Manufacturing Variation and Drip Irrigation Uniformity

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Manufacturing Variation and Drip Irrigation Uniformity

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ABSTRACT

THE coefficient of variation was used to assess the magnitude of emitter flow variation along single chamber drip irrigation lateral lines. The relationship between hydraulic variation and manufacturing variation was found to be orthogonal. This independent relationship allows the statistical combination of their respective coefficients of variation. As a result the statistical uniformity coefficient was recommended for use in determining the drip irrigation lateral line design uniformity including manufacturing variation.

INTRODUCTION

The purpose of drip irrigation is to apply water to the base of plants in frequent low volumes in an attempt to meet the consumptive use of plants. With this purpose in mind, it is essential that the emitter flow variation and/or the uniformity of water distribution be known, particularly since irrigation time and rate are ultimately based upon these variables.

The design of single chamber drip irrigation lateral lines considering hydraulic variation has been presented by various researchers. Myers and Bucks (1972) and Wu and Gitlin (1974, 1975) derived the hydraulic energy gradient line for determining the emitter flow variation and uniformity along a lateral line. Howell and Hiler (1974 a,b) used the hydraulic energy gradient principle and developed lateral line design equations based upon specific uniformity criteria.

Once the hydraulic design has been achieved, the acceptability of the design with respect to manufacturing and other variation must be determined. Keller and Karmeli (1974) introduced the coefficient of variation as a statistical measure for emitter manufacturing variation. This coefficient of manufacturer's variation was then included in design equations for emission uniformity. Solomon (1979, 1977) determined the manufacturing variation for various single and multiple orifice type emitters. Nakayama, et al. (1978) proposed a method based upon the coefficient of variation, for relating the number of emitters per plant to the application uniformity.

This paper is an attempt to statistically include manufacturing variation in the calculations for uniformity and emitter flow variation of single chamber drip irrigation lateral lines. The theoretical development and experimental results were originally presented in an M.S. thesis by Bralts (1978).

THEORETICAL DEVELOPMENT

The theoretical relationship between hydraulic variation and manufacturing variation was approached by, first, analyzing the individual components of emitter flow variation and, then, consolidating the variables into a single design equation.

Hydraulic Variation of Emitter Flow

In general, the equation for drip irrigation emitter flow has been shown by Wu and Gitlin (1974), Howell and Hiler (1974 a,b) and Karmeli (1977) to be

$$q = kh^x \qquad [1]$$

where

q is the emitter flow,
k is the constant flow of proportionality,
h is the pressure head at the emitter, and
x is the emitter discharge exponent.

The standard statistical equation for the variance using emitter flow (q) as the random variable is,

$$S_q^2 = \frac{1}{n} \sum_{i=1}^{n} (q_i - \bar{q})^2 \qquad [2]$$

where

S_q^2 is the variance of the random variable q,
n is the total number of emitters,
i is the subscript identifying a particular emitter,
q_i is the emitter flow, and
$\bar{q}$ is the mean emitter flow.

Because it is more expedient to calculate the variance directly from the individual emitter flows without first calculating the mean flow ($\bar{q}$), the following alternate form for the variance equation was used.

$$S_q^2 = \frac{1}{n} \sum_{i=1}^{n} q_i^2 - \frac{1}{n^2} \left(\sum_{i=1}^{n} q_i \right)^2 \qquad [3]$$

Next, substituting the components of emitter flow (kh^x) from equation [1] into the variance of emitter flow equation [3] results in

$$S_q^2 = \frac{1}{n} \Sigma (k_i h_i^x)^2 - \frac{1}{n^2} \left(\sum_{i=1}^{n} k_i h_i^x \right)^2 \qquad [4]$$

The above equation is the general form for calculating the variance of q when given the components of the individual emitter flow equations.

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At this point it is useful to examine the emitter flow equation [1] more closely. The k term, or constant of proportionality, includes many factors which deal with emitter construction such as the coefficient of discharge and cross sectional area of orifice type emitters. We therefore assumed that any variation of emitter flow due to manufacturing would be found in k. The h term, or pressure head, and the x term or emitter discharge exponents on the other hand, are hydraulically dependent variables. Thus any variation of emitter flow due to hydraulics was assumed to be found in the values of h and x where x is the assumed constant for any specific emitter type. With these assumptions the hydraulic and manufacturing components of emitter flow variation can be derived.

The variation of emitter flow along a lateral line due to frictional and elevational hydraulic pressure differences can be calculated by fixing k of the emitter flow equation. Substituting this constant into equation [4] and removing it from the summation we find

$$S_q^2 = k^2 \left[\frac{1}{n} \sum_{i=1}^{n} (h_i^x)^2 - \left(\frac{1}{n^2} \sum_{i=1}^{n} h_i^x\right)^2\right] \quad [5]$$

or

$$S_q^2 = k^2 \, S_{HL}^2 \quad [6]$$

where S_{HL}^2 is defined as the variance of emitter flow due to the hydraulics of the lateral line. Then taking the square root of S_q^2 and dividing by the mean emitter flow $\overline{q}$ one obtains

$$V_{HL} = \frac{\sqrt{S_q^2}}{\overline{q}} = \frac{S_{HL}}{\overline{h^x}} \quad [7]$$

where

V_{HL} is the coefficient of variation of emitter flow due to lateral line hydraulics

S_{HL} is the standard deviation of emitter flow due to hydraulics, and

$\overline{h^x}$ is the mean value of h^x.

A simple hydraulic method of estimating the emitter flow (q_i) along a lateral line was developed by Wu and Gitlin (1974). This method was based upon a dimensionless energy gradient line and is represented by the following equation

$$h_i = H_o - R_i \, \Delta H \pm R_i' \, \Delta H' \quad [8]$$

where

h_i is the pressure at the emitter,

H_o is the original pressure,

R_i is the friction energy drop ration or [$R_i = 1 - (1 - i)^{2.852}$] for the Williams and Hazen equation,

ΔH is the total energy drop due to friction,

R_i' is the elevational energy drop ration or [$R_i' = i$] for uniform slope,

$\Delta H'$ is the total energy gain or loss due to elevation, and

i is the subscript denoting specific emitters at lateral line positions ℓ/L.

A plot of this relationship is shown in Fig. 1.

The emitter flow at any point can be calculated by substituting the above quation into the emitter flow equation [1] and results in

$$q_i = k[H_o - R_i \, \Delta H \pm R_i' \, \Delta H']^x \quad [9]$$

Next we can remove k by dividing the above equation by the relationship ($q_o = kH_o^x$) for the orifice flow at H_o which yields

$$q_i = q_o \left[1 - \frac{R_i \, \Delta H}{H_o} + \frac{R_i' \, \Delta H'}{H_o}\right]^x \quad [10]$$

Thus given the emitter flow at the operating pressure H_o and the energy gradient line, the individual emitter flows can be calculated independently of the constant of proportionality, k. When these values are subsequently used in equation [3] and [7], the coefficient of variation of emitter flow due to lateral line hydraulics (V_{HL}) results.

Manufacturing Variation of Emitters

The manufacturers' variation of emitters (V_m) was first developed by Keller Karmeli (1974). The technique used to find the manufacturing variation included measuring the emitter flow under constant head or pressure. If the pressure h and the emitter discharge exponent x of the emitter flow variation equation [1] are held constant, then the variance can be calculated using equation [4]. Removing the constant h^x from the summation we find

$$S_q^2 = h^{2x} \left[\frac{1}{n} \sum_{i=1}^{n} (k_i)^2 - \frac{1}{n^2} \left(\sum_{i=1}^{n} k_i\right)^2\right] \quad [11]$$

or

$$S_q^2 = h^{2x} \, S_{ME}^2 \quad [12]$$

where s_{ME}^2 is defined as the variance of emitter flow due to manufacturing. Then, taking the square root of S_q^2 and dividing by the mean $\overline{q}$ results in

$$V_{ME} = \frac{\sqrt{S_q^2}}{\overline{q}} = \frac{S_{ME}}{\overline{k}} \quad [13]$$

where

V_{ME} is the coefficient of variation of emitter flow due to emitter manufacturing and

S_{ME} is the standard deviation of emitter flow due to manufacturing.

Thus, given the individual emitter flows at a constant pressure, these values can be used in equation [3] and [13] to calculate the coefficient of variation due to emitter manufacturing.

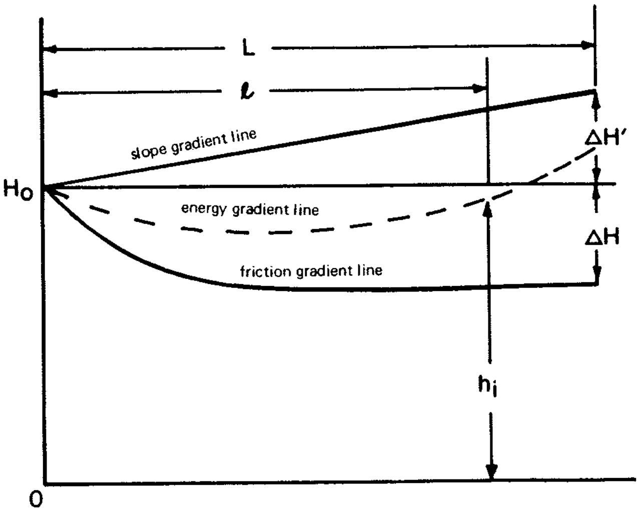


FIG. 1 Lateral line notation.

Combined Lateral Line Emitter Flow Variation

In basic statistics if any two variables are independent contributors to the variance of a third variable, then their variances can be combined. Beginning with the factors affecting lateral line emitter flow variation just described, two independent variables hydraulics h^x and manufacturing k logically emerge. The assumption behind the independence of these two components derives from the knowledge that the variance due to hydraulics h^x is caused by friction in the pipe, and that the variance due to manufacturing k is caused by cross sectional area changes and coefficient of discharge changes for the individual emitters. This logic is intuitively based and, hence, some proof of its validity in terms of experimental data will be required.

Continuing, however, with the assumption that h^x and k are independent, the two variances can be combined as moments of a product. Using equations [6] and [12] for the variance of q due to hydraulics and q due to manufacturing, respectively, and the equation for total variance of the lateral line (S^2_{TL}) becomes

$$S^2_{TL} = k^2\, S^2_{HL} + h^{2x}\, S^2_{ME} + k^2\, S^2_{HL}\, h^{2x}\, S^2_{ME} \quad \ldots\ldots\ [14]$$

Furthermore, equation [14] can be standardized into the coefficients of variation by first dividing both sides of equation [14] by the $\bar{q}$ squared and then taking the square root. This yields a simplified equation for the total lateral line coefficient of emitter flow variation (V_{TL}).

$$V_{TL} = \sqrt{V^2_{HL} + V^2_{ME} + V^2_{HL}V^2_{ME}} \quad \ldots\ldots\ [15]$$

Given the coefficients of variation due to hydraulics as calculated by equation [7] and manufacturing variation as calculated by equation [13] the coefficient of total lateral line emitter flow variation can be found.

For coefficients of variation less than 0.2, as found in most drip irrigation applications, the last term of equation [15] can be neglected. In such instances the statistical application of the Pythagorean theorem is appropriate and can be illustrated in equation [16] and Fig. 2.

$$V_{TL} = \sqrt{V^2_{HL}\ V^2_{ME}} \quad \ldots\ldots\ [16]$$

Using this simplified relationship, the lateral line emitter flow variation or uniformity can be more easily determined. The following is an example of the use of the statistical uniformity equation

$$U_s = 100\,(1 - V_{TL}) \quad \ldots\ldots\ [17]$$

The above equation is a statistically based estimate of emitter flow variation for drip irrigation lateral lines. The advantage of this equation over the other presently used uniformity measures is that additional factors such as emitter plugging lateral line temperature variation and other variations may eventually be included in the final uniformity coefficient.

EXPERIMENTAL TESTING AND RESULTS

The following is a description of the experimental apparatus, experimental testing and results.

Experimental Apparatus

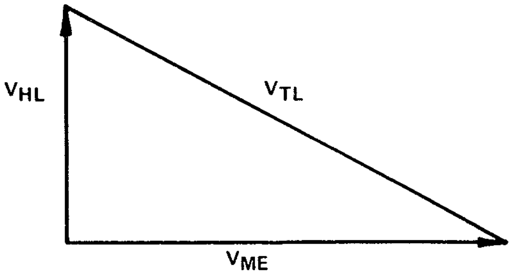


FIG. 2 The statistical application of the Pythagorean theorem.

The experimental testing was performed on a specially built drip irrigation Hydraulic Design Test Stand (HDTS). The HDTS is simply a mechanized emitter flow collection device. Structurally the HDTS consists of a single row of 100, 30 cm × 15 cm rectangular funnels supported by a frame of steel angle iron. Below each funnel is a shop modified 1,000 mL graduated cylinder which is pneumatically controlled for individual emitter flow collection. The hydraulic controls of the HDTS consisted of inflow and outflow rate meters, pressure gauges and differential manometers (Fig. 3). For this experiment the HDTS slope was kept at zero percent.

With the HDTS, 30 m sections of single chamber lateral lines were tested for emitter flow variation and lateral line hydraulics with relative ease. In addition, since both inflow and outflow rates could be controlled, long length simulations were possible.

Experimental Testing

The general experimental approach was to design a test or group of tests which would confirm or reject specific theoretical relationships. Table 1 is a summary of the basic theoretical relationships and the required testing. Note the actual testing has been grouped into two categories: I. Basic Hydraulics, and II. Long Length Simulations. Table 2 outlines the testing performed under each of the categories.

The experimental tests conducted in Group I were designed to yield basic hydraulic and general emitter flow variation data. Random samples of single chamber drip

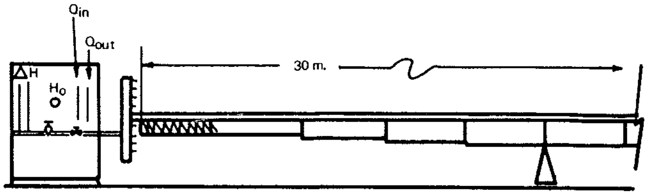


FIG. 3a HDTS structural framework.

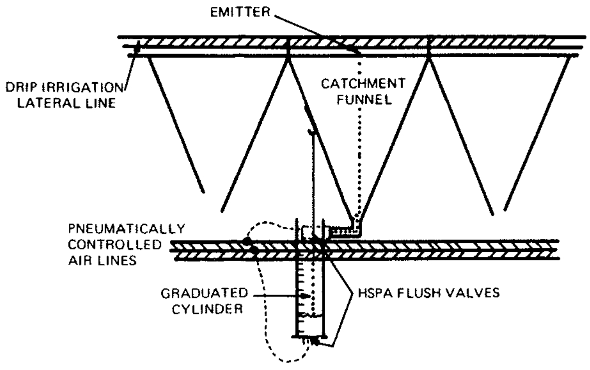


FIG. 3b HDTS emitter flow collection device.

TABLE 1. EXPERIMENTAL DESIGN

Group	Theoretical equations	Required testing	Group
I Basic hydraulics (1) Hydraulic variation of emitter flow	$V_{HL} = \frac{\sqrt{S_q^2}}{\bar{q}} = \frac{S_{HL}}{\overline{h^x}}$	[7] Emitter flow variation data with constant k value is required but not feasible. However, ΔH can be measured under various hydraulic conditions. The energy gradient line can then be calculated and the hydraulic variation estimated.	I
(2) Manufacturing variation of emitter flow	$V_{ME} = \frac{\sqrt{S_q^2}}{\bar{q}} = \frac{S_{ME}}{\bar{k}}$	[13] Emitter flow variation data at several constant pressures is required.	I
(3) Independence assumption	$V_{TL}^2 = V_{HL}^2 + V_{ME}^2 - 2V_{HL}V_{ME}\cos\theta$	[18] Emitter flow variation data of various manufacturing is required to show that hydraulic and manufacturing emitter flow variation are independent for that $\theta = 90°$.	I
II Long length simulations (1) Total variation	$V_{TL} = \sqrt{V_{HL}^2 + V_{ME}^2}$	[16] Emitter flow variation data for long lengths is required, to demonstrate the validity of the simplified equation for long lengths.	II

irrigation lateral lines were obtained and put through a series of hydraulic tests on the HDTS. More specifically, the results of this group of testing should confirm or reject the assumption of orthogonality of hydraulic variation and manufacturing variation, and, in so doing, demonstrate the validity of the theoretically derived statistical relationships given in the previous section.

In Group I the experimental testing of the variation of emitter flow due to hydraulic variation was not measured directly because, to do so, a lateral line with a uniform emitter manufacturing variation was needed. As part of the hydraulic testing data, the lateral line head loss ΔH was measured, then the hydraulic variation or the coefficient of hydraulic variation of the lateral line (V_{HL}) was calculated using the basic Hydraulic equations shown in the theoretical development.

The manufacturing variation of emitters for individual lateral lines was determined by assuming the head to be constant over the last 50 percent of the lateral line. (Fig. 1 demonstrates the validity of this assumption.) Then the emitter flows over the last 50 percent of the lateral line were used to calculate the coefficient of variation due to emitter manufacturing (M_{ME}).

The Group II experimental testing was designed to extend the relationships developed from the 30 m lateral lines to 150 m lateral lines. This extension would satisfy the maximum normal field length for lateral lines. To do this the inflow and outflow controls of the HDTS were used to individually simulate 30 m sections of a 150 lateral line.

Results

The manufacturing variation of emitters was assumed in the theoretical development to be independent of pressure when expressed as the coefficient of manufacturing variation of emitters (V_{ME}). To demonstrate the validity of this assumption, the V_{ME} versus h curves were drawn. Fig. 4 shows that the predicted independence of manufacturing variation holds very well for the single chamber tubing.

Since the degree of correlation between the coefficient of manufacturing variation of the emitter (V_{ME}) and the coefficient of hydraulic variation of the lateral line (V_{HL}) for the data was unknown, orthogonality or complete independence was assumed. To demonstrate the validity of this assumption the real angle (Θ) between V_{ME} and V_{HL} must be determined. Using the Group I analysis results and the law of cosines we find:

$$V_{TL}^2 = V_{HL}^2 + V_{ME}^2 - 2V_{HL}\,V_{ME}\cos\theta \qquad [18]$$

where V_{TL} is the total coefficient of variation found for the lateral. When actually calculating the individual angles for various tests, the angles vary from 75 deg. to 105 deg. To clarify the results, a composite vector was calculated for the Group I tests. The graphs and angles are shown in Fig. 5.

The results demonstrate decisively that the hydraulic and manufacturing variation for single chamber tubes are orthogonal ($\Theta = 90$ deg.), thus supporting our original assumption of independence.

In the Group I analysis the actual coefficients of total variation found (V_{TF}) were determined. Then using the calculated coefficients of hydraulic variation (V_{HL}) and the actual coefficients of manufacturing variation (V_{ME}) the calculated coefficients of total variation (V_{TC}) were determined using equation [16]. The results were then graphed as V_{TF} versus V_{TC} and are shown for single tubing in Fig. 6.

Continuing with assumption of independence of hydraulic and manufacturing variation, several long length simulations (150 m) using the HDTS, were performed to

TABLE 2. EXPERIMENTAL TESTING

	Number of samples	Number of tests	Type of tube	Length simulated, m	Pressures, head, m	Outflow, L/s
Group I	8	24	Single chamber	30	2.1, 4.2, 8.4	0
(Basic hydraulics)	8	24	Single chamber	30	2.1, 4.2, 8.4	⩾0
Group II (Long length stimulation)	3	15	Single chamber	150	2.1 and up	⩾0

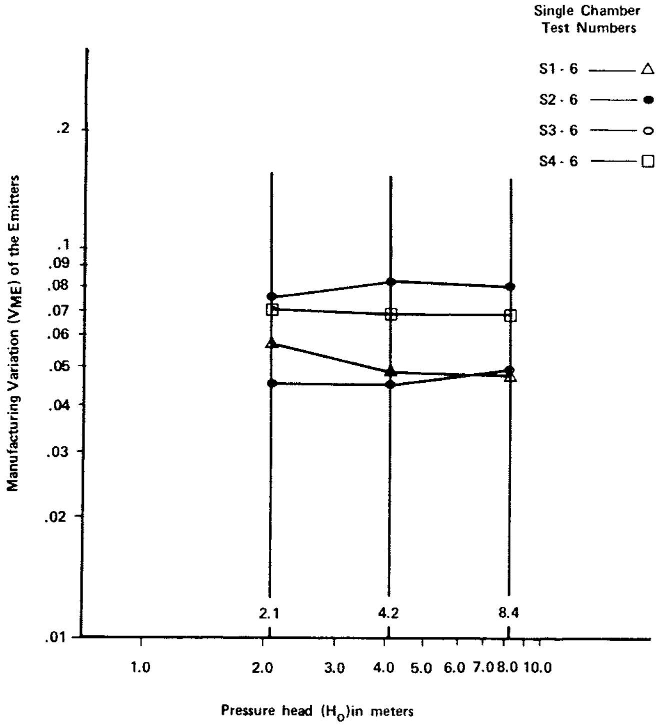


FIG. 4 Single chamber tubing coefficient of manufacturing variation versus pressure.

show the validity of equation [16] under approximate field conditions.

$$V_{TL} = \sqrt{V_{HL}^2 + V_{ME}^2} \quad [19]$$

The results were analyzed and then graphed as the coefficient of total variation found versus the coefficient of total variation calculated (Fig. 7).

Discussion

From the preceding results, hydraulic and manufacturing variation were shown to be relevant in the design of single chamber drip irrigation lateral lines. Since single chamber lateral emitters are specific type emitters, the general applicability of these results to other types of specific emitters is possible.

Dual chamber tubing was tested on the HDTS in a similar fashion as the single chamber tubing. The results, however, were not as conclusive. This is because dual chamber lateral emitters are not specific type emitters and thus the method of testing inadvertently included the hydraulic variation caused by varying lengths of flow paths in the secondary chamber.

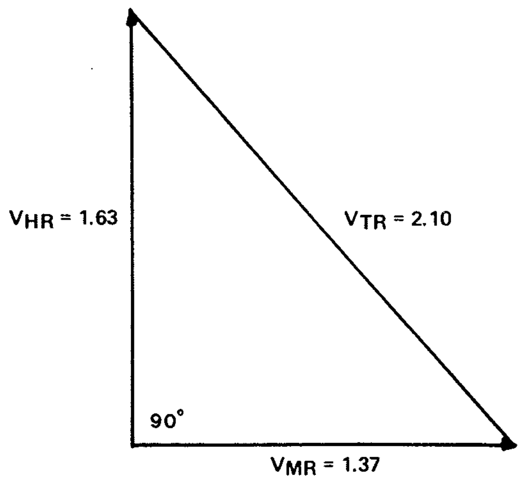


FIG. 5 Graphic demonstration of the vector relationship between the coefficients of hydraulic and manufacturing variation.

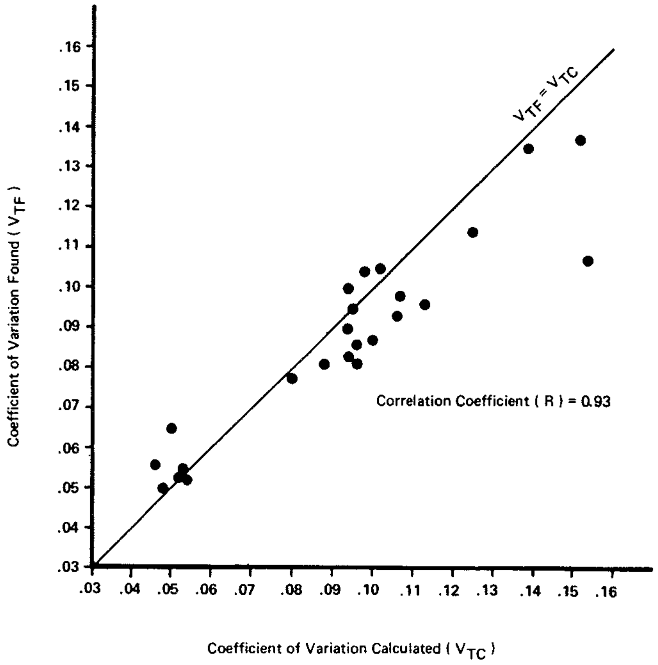


FIG. 6 Coefficients of total variation found versus total coefficient of variation calculated for single chamber tubing.

ENGINEERING APPLICATIONS

To the drip irrigation design engineer, it is essential that the above theory, results, and analyses be translated into some form of useful design procedure. In this section several simplified design charts and nomographs will be presented along with an example design.

Hydraulic Design Charts

The hydraulic design charts and nomographs shown in Figs. 8 and 9 are a simple modification of the

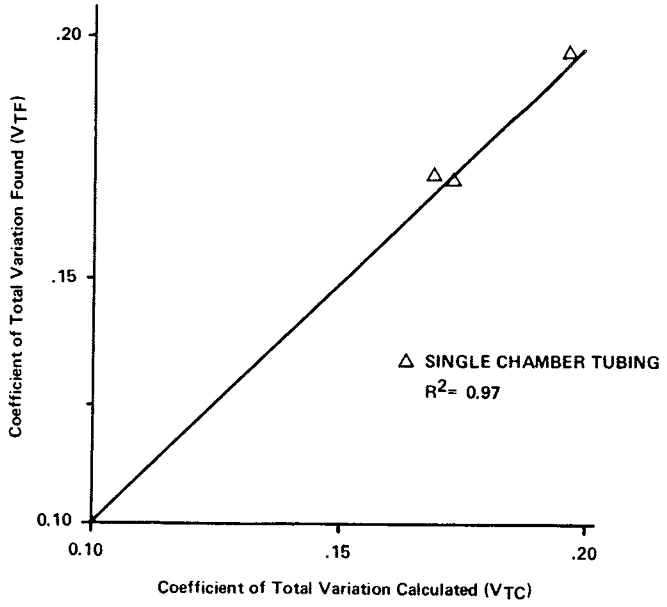
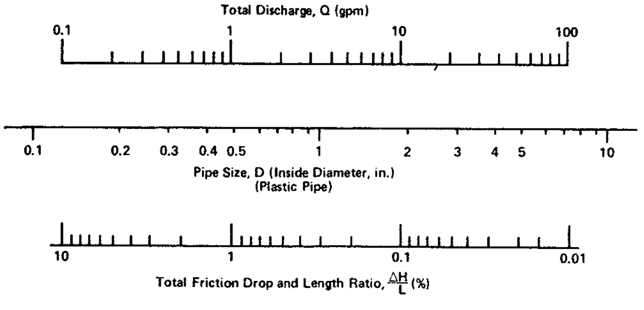
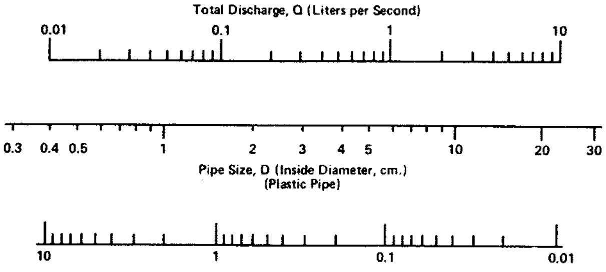


FIG. 7 Single chamber long length simulation coefficients of total variation found versus calculated.



Nomograph for Drip Irrigation Laterals and Submain Design in British Units



Nomograph for Drip Irrigation Laterals and Submain Design in Metric Units

FIG. 8 Nomograph for total friction drop (ΔH) and length (L) ratio, ΔH/L (%).

nomographs and four quadrant design charts developed by Wu and Gitlin (1977). The only change involves the conversion of quadrant I of the design chart to the statistical uniformity U_s as defined by equation [17]. This change enables the inclusion of other factors such as manufacturing variation in the final uniformity calculations. The basis of the design chart is equation [10] of this paper and the procedure for using the drip irrigation hydraulic design charts in metric form (Figs. 8 and 9) can be found in the publication by Wu (1977).

Manufacturing Variation Nomograph

In order to simplify the inclusion of manufacturing variation into the overall drip irrigation design procedures, a nomograph solving for equation [16] was developed. Thus, given the hydraulic variation of uniformity as calculated from the hydraulic design charts and the manufacturing variation, the resulting emitter flow variation or uniformity including manufacturing can be found. Fig. 10 is an example of this nomograph using the statistical uniformity.

Design Example

The following is an example of drip irrigation lateral line design considering hydraulic and manufacturing variation.

Given: A 1.27 cm (0.5 in.) lateral line 152 m (500 ft) long that is laid on a 2 percent down grade. The input pressure head is 7 m (231.1 ft) and the total discharge is estimated at 0.13 l/s (2 gpm). The coefficient of manufacturing variation of the emitters as given by the manufacturer is 10 percent.

Required: Determine the statistical uniformity of the above lateral line.

Solution:

1 Beginning with Figs. 8 and 9, determine the hydraulically caused statistical uniformity. (Ans. U_s = 95 percent).

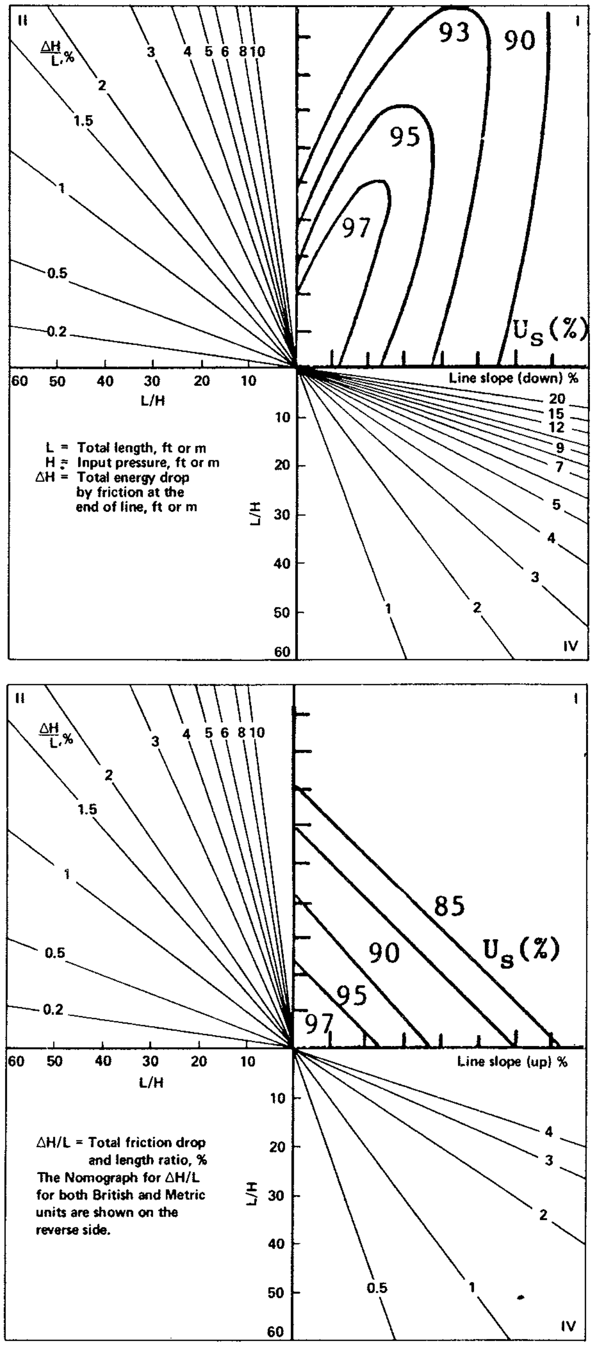


FIG. 9 Drip irrigation design calculator using statistical uniformity.

2 Using the above result and Fig. 10, calculate the statistical uniformity including the manufacturing variation. (Ans. U_s = 89 percent).

SUMMARY AND CONCLUSIONS

This study has shown that hydraulic and manufacturing variation can be statistically combined and included in the design equations for uniformity of single chamber drip irrigation lateral lines. The inclusion of variation due to emitter manufacturing enables the design engineer to assess the ramifications of manufacturing variation on uniformity, irrigation application rates, and, ultimately, crop yields.

The experimental testing tended to confirm the theoretically predicted relationships for single chamber tub-

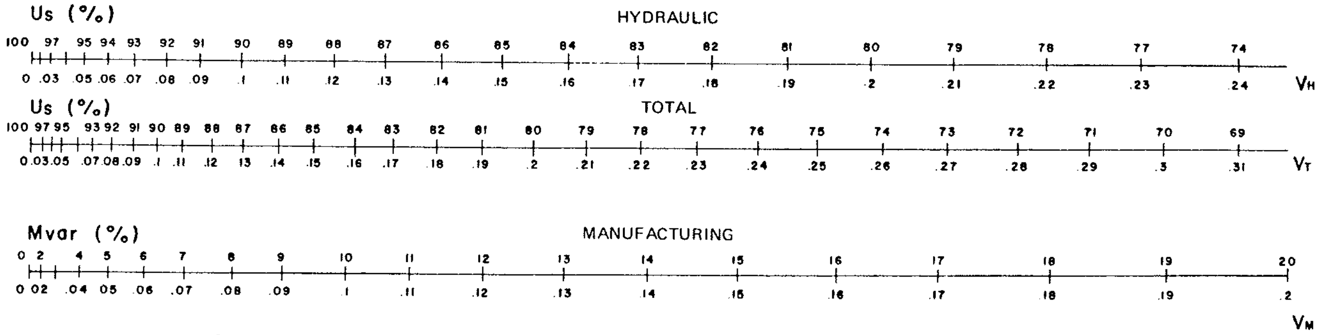


FIG. 10 Nomograph for calculating the statistical uniformity given the hydraulic and manufacturing variation.

ing. In addition, the extension of these results to other specific type emitters is possible.

Design curves and nomographs have been developed to simplify the inclusion of emitter manufacturing in the design of drip irrigation lateral lines. The theory presented, however, reaches beyond the simple hydraulic and manufacturing variation. Variations of emitter flow due to water temperature, emitter plugging and emitter spacing can also be statistically included in the uniformity and emitter flow variation equations presented.

In conclusion, a step toward a statistically based method of evaluating drip irrigation emitter flow variation has been developed. The emitter manufacturing variation was included in the uniformity estimates and design procedures for drip irrigation lateral line systems.

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The following symbols are used in this paper:

h = pressure head at the emitter
H_o = pressure at the head of the lateral line
ΔH = head loss in a lateral line due to friction
$\Delta H'$ = head gain or loss in a lateral line due to slope
k = constant of proportionality for emitter flow
ℓ = lateral line length to a specific emitter
L = total lateral line length
q = emitter flow
$\bar{q}$ = mean emitter flow
q_o = emitter flow of the first emitter
R_i = friction energy drop ratio
R_i' = elevation energy drop ratio
S_{HL} = standard deviation of emitter flow due to hydraulics
S_{ME} = standard deviation of emitter flow due to manufacturing
S^2_{HL} = variance of emitter flow due to lateral line hydraulics
S^2_{ME} = variance of emitter flow due to manufacturing
S^2_q = variance of emitter flow
S^2_{TL} = total variance of emitter flow along the lateral line
U_s = statistical uniformity
V_{HL} = coefficient of variation of emitter flow due to hydraulics
V_m = coefficient of manufacturers' variation
V_{ME} = coefficient of variation of emitter flow due to manufacturing
V_{TL} = total coefficient of variation of emitter flow along the lateral line
x = emitter discharge exponent
θ = angle between the vectors for manufacturing and hydraulic variation

Subscripts

i = integer subscripts for all emitters
n = total number of emitters