\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \]

\[ b_n = \frac{1}{\sqrt{n}} \]

\[ \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0 \]

\[ b_n = \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} = b_{n+1} \]

MATE 3032

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UPRM
Estrategias para pruebas de series

En las secciones anteriores se han presentado algunos métodos para estudiar la convergencia o divergencia de una serie. La decisión es determinar que prueba se utiliza para probar la convergencia o divergencia. Algunas sugerencias:

1. If the series is of the form \( \sum \frac{1}{n^p} \), it is a p-series, which we know to be convergent if \( p > 1 \) and divergent if \( p < 1 \).
2. If the series has the form \( \sum ar^{n-1} \) or \( \sum ar^n \), it is a geometric series, which converges if \( |r| < 1 \) and diverges if \( |r| \geq 1 \). Some preliminary algebraic manipulation may be required to bring the series into this form.
3. If the series has a form that is similar to a p-series or a geometric series, then one of the comparison tests should be considered. In particular, if \(a_n\) is a rational function or an algebraic function of \(n\) (involving roots of polynomials), then the series should be compared with a p-series. The comparison tests apply only to series with positive terms, but if \(\sum a_n\) has some negative terms, then we can apply the Comparison Test to \(\sum |a_n|\) and test for absolute convergence.

4. If you can see at a glance that \(\lim_{n \to \infty} a_n \neq 0\) then the Test for Divergence should be used.

5. If the series is of the form \(\sum (-1)^{n-1} b_n\) or \(\sum (-1)^n b_n\), then the Alternating Series Test is an obvious possibility.

6. Series that involve factorials or other products (including a constant raised to the nth power) are often conveniently tested using the Ratio Test. Bear in mind that \(\left|\frac{a_{n+1}}{a_n}\right|\) as \(n \to \infty\) for all p-series and therefore all rational or algebraic functions of \(n\). Thus the Ratio Test should not be used for such series.
7. If \( a_n \) is of the form \((b_n)^n\), then the Root Test may be useful.

8. If \( a_n = f(n) \), where \( \int_1^\infty f(x) \, dx \) is easily evaluated, then the Integral Test is effective (assuming the hypotheses of this test are satisfied).

Ejemplo

1. 2 pág. 746

\[
\lim_{n \to \infty} \frac{b_n}{c_n} = \lim_{n \to \infty} \frac{n-1}{n^3+1} = \lim_{n \to \infty} \frac{n-1}{n^3} = \lim_{n \to \infty} \frac{(n^3-h^n-2)/h^3}{n^3+1} = 1
\]

2.4 pág. 746

\[
\sum_{n=1}^{\infty} (-1)^n \left( \frac{n-1}{n^2+1} \right) \quad \text{serie alternada} \quad D
\]

\[
\lim_{n \to \infty} \frac{n^2-1}{n^2+1} = 1 - j
\]
3. 6 pág. 746

\[ \sum_{n=1}^{\infty} \frac{n^{2}h}{(n+1)^3} = \sum_{n=1}^{\infty} \left[ \frac{n^2}{(n+1)^3} \right] h \]

Aplicando criterio ras1:

\[ \lim_{n \to \infty} n \sqrt{n} = \lim_{n \to \infty} \sqrt{\left( \frac{n^2}{(n+1)^3} \right)} \]

\[ = \lim_{n \to \infty} \frac{n^2}{(n+1)^3} = 0 < 1 \]
4. 10 pág. 746

\[ \sum_{n=1}^{\infty} \frac{n^2 e^{-n^3}}{a_n} \leq f(x) = x^2 e^{-x^3} \]

\[
\int_{1}^{\infty} x^2 e^{-x^3} \, dx = \lim_{b \to \infty} \left[ -\frac{1}{3} e^{-x^3} \right]_1^b
\]

\[ u = -x^3 \quad \Rightarrow \quad du = -3x^2 \, dx \]

\[
= \lim_{b \to \infty} -\frac{1}{3} \left[ e^{-b^3} - e^{-1} \right] = \frac{1}{3e}
\]
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\[
\lim_{n \to \infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n-1)}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}
\]

\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2n-1) \cdot (2n+1)}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1) \cdot (3n+2)}
\]

\[
= \lim_{n \to \infty} \frac{2n+1}{3n+2} = \frac{2}{3} < 1
\]
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\[
\lim_{k \to \infty} \frac{\frac{1}{2+\sin k} \sum_{k=1}^{N+1}}{a_k}
\]

\[C \neq D?\]

Com o: \(-1 < \lim_{k \to \infty} \sin k < 1\)

\[\lim_{k \to \infty} \frac{1}{2+\sin k} \neq 0\]
6. 25 pág. 746

\[
\begin{align*}
\lim_{h \to \infty} \frac{G_{n+1}}{G_n} &= \lim_{h \to \infty} \frac{(n+1)!}{e^{2h+1}} \\
&= \lim_{h \to \infty} \frac{(n+1)!}{e^{2h+1}} \\
&= 1 \quad (n+1)! > 2e^{2h+1}
\end{align*}
\]