DYNAMICS OF A PARTICLE: Beer and Johnston 2004

Chapter 11
Kinematics
(Geometry)

Vel. & acc. in rectilinear motion:
$v = dx/dt \quad a = dv/dt = d^2x/dt^2 = v\, dv/dx$

Relative motion of 2 particles:
$x_B = x_A + x_{B/A}$
$v_B = v_A + v_{B/A}$
$a_B = a_A + a_{B/A}$

Rect. components of vel. & acc.:
$v_x = x \quad v_y = y \quad v_z = z$
$a_x = x \quad a_y = y \quad a_z = z$

Tangencial & normal components:
$\mathbf{v} = v\mathbf{e}_t$
$\mathbf{a} = dv/dt\, \mathbf{e}_t + v^2/\rho\, \mathbf{e}_n$

Uniform rect. motion:
$x = x_0 + vt$

Uniformly accelerated rect. motion:
$v = v_0 + at$
$x = x_0 + v_0t + ½\, at^2$
$v^2 = v_0^2 + 2a(x - x_0)$

Position & vel. in curvilinear motion:
$\mathbf{v} = d\mathbf{r}/dt \quad v = ds/st$

Accel in curvilinear motion: $a = dv/dt$

Relative motion of 2 particles :
$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Chapter 12
Kinetics
(F = ma)

Newton's Secon Law: $\Sigma\mathbf{F} = m\mathbf{a}$

Equations of Motion for a Particle :
$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y \quad \Sigma F_z = ma_z$
$\Sigma F_t = m\, dv/dt \quad \Sigma F_n = m\, v^2/\rho$

Rate of change of ang momentum : $\Sigma\mathbf{M}_0 = \mathbf{H}_0$

Radial & transverse components:
$\Sigma F_r = m(r - r\Theta^2)$
$\Sigma F_\theta = m(r\Theta - 2r\Theta)$

Motion under central force: $\mathbf{H}_0$ = constant
$rmv \sin\varphi = = r_0mv_0 \sin\varphi_0 \quad r^2\,\Theta = h$

Orbital motion :
$(d^2u/d\Theta^2) + u = F/mh^2u^2 \quad 1/r = u = (GM/h2) + C\cos\Theta$

Escape velocity:
$v_{esc} = \sqrt{(2GM/r_0)} \quad v_{circ} = \sqrt{(GM/r_0)}$

Linear Momentum: $\mathbf{L} = m\mathbf{v} \quad \Sigma\mathbf{F} = \mathbf{L}$

Dynamic equilibrium:
$\mathbf{H}_0 = \mathbf{r} * m\mathbf{v}$
$H_0 = rmv \sin\varphi$

$$H_0 = \begin{vmatrix} i & j & k \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

$H_0 = H_z = m(xv_y - yv_x)$

Newtons law of universal gravitation:
$F = G\,Mm/r^2 \quad GM = gR^2$

Periodic Time:
$\tau = 2\pi ab/h$

Chapter 13
Work & Energy

Work force: $dU = \mathbf{F}\cdot d\mathbf{r} \quad dU = F\, ds \cos\alpha \quad U_{1-2} = {}_{A1}\int^{A2} F\cdot dr = {}_{A1}\int^{A2} (F_xdx + F_ydy + F_zdz)$

Work of weight: $U_{1-2} = -{}_{y1}\int^{y2} W dy = W_{y1} - W_{y2}$

Work of force by spring: $dU = -F\,dx = -kxdx \quad U_{1-2} = -{}_{x1}\int^{x2} kx\, dx = ½\, kx - ½\, kx$

Work gravity force: $U_{1-2} = -{}_{r1}\int^{r2} (G\,Mm/r^2)dr = (G\,Mm/r_2) - (G\,Mm/r_1)$

Kinetic energy of particle: $T = ½\, mv^2$

Power & mechanical efficiency:
Power $= dU/dt = \mathbf{F}\cdot\mathbf{v}$
η = power output / power input

Principle of work & energy : $T_1 + U_{1-2} = T_2$

Potential energy: $U_{1-2} = V_2 - V_1$
(weight) $V_g = Wy$ (gravitational) $V_g = -G\,Mm/r$
(elastic-spring) $V_e = ½\, kx^2 \quad T_1 + V_1 = T_2 + V_2$

Motion under gravity force:
$(H_0)_0 = H_0$: $rmv \sin\varphi = = r_0mv_0 \sin\varphi_0$
$T_0 + V_0 = T + V$: $½\, mv^2 - G\,Mm/r_0 = ½\, mv^2 - G\,Mm/r$

Impulse & momentum for particle:
$m\mathbf{v}1 + {}_{t1}\int^{t2} \mathbf{F}dt = m\mathbf{v}_2$
$m\mathbf{v}1 + \mathbf{Imp}_{1-2} = m\mathbf{v}_2$
$m\mathbf{v}1 + \Sigma\mathbf{Imp}_{1-2} = m\mathbf{v}_2$

Direct central impact:
$m_Av_A + m_Bv_B = m_Av'_A + m_Bv'_B$
$v'_B - v'_A = e\,(v_A + v_B)$

Impulsive motion:
$m\mathbf{v}_1 + \Sigma\mathbf{F}\,\Delta t = m\mathbf{v}_2$
$\Sigma m\mathbf{v}_1 + \Sigma\mathbf{F}\,\Delta t = \Sigma m\mathbf{v}_2$

DYNAMICS OF A RIGID BODY: Beer and Johnston 2004

Chapter 15 Kinematics of Rigid Bodies	**Rigid Body in Rotation about a fixed axis:** $v = ds/dt = r\Theta \sin\varphi_0$ $\mathbf{v} = d\mathbf{r}/dt = \boldsymbol{\omega} \times \mathbf{r}$ $\boldsymbol{\omega} = \omega\mathbf{k} = \Theta\mathbf{k}$ **acceleration:** $\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ **angular acc:** $\boldsymbol{\alpha} = \alpha\mathbf{k} = \omega\mathbf{k} = \Theta\mathbf{k}$ **Rotation of a rep. slab:** **Tangencial & Normal components:** $\mathbf{v} = \omega\mathbf{k} \times \mathbf{r}$ $\mathbf{a}_t = \alpha\mathbf{k} \times \mathbf{r}$ $a_t = r\alpha$ **Angular vel & acc of rotating slab:** $\mathbf{a}_n = -\omega^2\mathbf{r}$ $a_n = r\omega^2$ $\omega = d\Theta/dt$ $\alpha = d\omega/dt = d^2\Theta/dt^2 = \omega\, d\omega/d\Theta$ **Velocities in plane motion:** $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\mathbf{v}_{B/A} = \omega\mathbf{k} \times \mathbf{r}_{B/A}$ $v_{B/A} = r\omega$ **Acc. in plane motion:** $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$ **Rate of change of vector with respect to rotatig frame** : $(\mathbf{Q})_{OXYZ} = (\mathbf{Q})_{Oxyz} + \boldsymbol{\Omega} \times \mathbf{Q}$ **Plane Motion of particle relative to rotating frame:** $\mathbf{v}_P = \mathbf{v}_{P1} + \mathbf{v}_{P/\mathcal{F}}$ $\mathbf{a}_P = \mathbf{a}_{P1} + \mathbf{a}_{P/\mathcal{F}} + \mathbf{a}_c$ **Motion of a Rigid Body with a fixed point:** $\mathbf{v} = d\mathbf{r}/dt = \boldsymbol{\omega} \times \mathbf{r}$ $\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ **General motion in space:** $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$ $\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A})$ **3-dimensional motion of particle rel to rotating frame:** $\mathbf{v}_P = \mathbf{v}_{P1} + \mathbf{v}_{P/\mathcal{F}}$ $\mathbf{v}_P$ – absolute vel of particle P $\mathbf{v}_{P1}$ – vel of point P^1 of moving frame $\mathcal{F}$ coinciding with P $\mathbf{v}_{P/\mathcal{F}}$ - vel of P relative to moving frame $\mathcal{F}$ $\mathbf{a}_P = \mathbf{a}_{P1} + \mathbf{a}_{P/\mathcal{F}} + \mathbf{a}_c$ $\mathbf{a}_P$ - absolute acc. of particle P $\mathbf{a}_{P1}$ - acc. of point P^1 of moving frame $\mathcal{F}$ coinciding with P $\mathbf{a}_{P/\mathcal{F}}$ - acc. of P relative to moving frame $\mathcal{F}$ $\mathbf{a}_c$ - $2\boldsymbol{\Omega} \times (\mathbf{r})_{Oxy} = 2\boldsymbol{\Omega} \times \mathbf{v}_{P/\mathcal{F}}$
Chapter 16 Plane Motion of Rigid Bodies (Forces & Acc.)	**Fundamental eqn. of motions for rigid body:** $\Sigma\mathbf{F} = m\mathbf{a}$ $\Sigma\mathbf{M}_G = \mathbf{H}_G$ **Angular momentum in plane motion:** $\mathbf{H}_G = I\boldsymbol{\omega}$ $\mathbf{H}_G = I\boldsymbol{\omega} = I\boldsymbol{\alpha}$ **Equations for the plane motion of a rigid body:** $\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$ $\Sigma M_G = I\alpha$
Chapter 17 Plane Motion of Rigid Bodies (Energy & Momentum)	**Principle of Work & Energy for a rigid body :** $T_1 + U_{1\text{-}2} = T_2$ **Work of a force or a couple:** $U_{1\text{-}2} = \int_{s1}^{s2} (F\cos\alpha)\, ds$ $U_{1\text{-}2} = -\int_{\theta 1}^{\theta 2} M\, d\theta$ **Kinetic energy in plane motion:** $T = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$ **Kinetic energy in rotation:** **Conservation of energy:** $T = \frac{1}{2} I_o\omega^2$ $T_1 + V_1 = T_2 + V_2$ **Power:** Power $= dU/dt = M\, d\theta/dt = M\omega$ **Principle of impulse & momentum for rigid body:** Syst Momenta $_1$ + Syst Ext Imp $_{1\text{-}2}$ = Syst Momenta $_2$ **Eccentric impact:** $(v^1_B)_n - (v^1_A)_n = e\,[(v_A)_n - (v_B)_n]$