What’s wrong with Hypothesis Testing?

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How I got interested in this topic

2011: Madrid - TEST2011 – Conference on the Null Hypothesis Significance Testing (NHST) controversy

2014 *Basic and Applied Social Psychology* (*BASP*) puts NHST “on probation”

2015 BASP bans use of NHST in any paper published

→ so, what’s wrong with hypothesis testing?

and no, it’s not (just) the usual Frequentist vs Bayesian thing

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An (Artificial) Example

Question: Is our undergraduate students intelligence the same as that in the general population?

Experiment: Randomly select 50 students and administer the **Wechsler Adult Intelligence Scale** (**WAIS**)

(scores from general population have a normal distribution with a mean of 10 and a standard deviation of 3)

Outcome: mean score of our students: 10.75.

So, does this prove that our students intelligence is different from the general population?

Textbook Solution

- Method: z-test
- Type I error prob. $\alpha = 0.05$
- $H_0 \colon \mu = 10$
- $H_a \colon \mu \neq 10$
- $\mathrm{Z} = \sqrt{n}\frac{\overline{X}-\mu}{\sigma} = \sqrt{50}\frac{10.75-10}{3} = 1.767$
- $p = 2\,P(N(0,1) > 1.767) = 0.077$
- $0.077 > 0.05$, fail to reject H_0

A brief history of HT

Idea has been around for a very long time:

John Arbuthnot (1710) H_0: God does not exist

Data: fraction of boys born is slightly larger than the fraction of girls

He calculated that assuming equal probabilities for boys and girls this empirical fact would be exceedingly unlikely:

p = 1/483600000000000000000000 (the first p-value!)

argued that this was a proof of God’s will - boys had higher risks of an early death

1925 – Sir Ronald A. Fisher “**Statistical** Method for Research Workers” (First Statistics Textbook!):

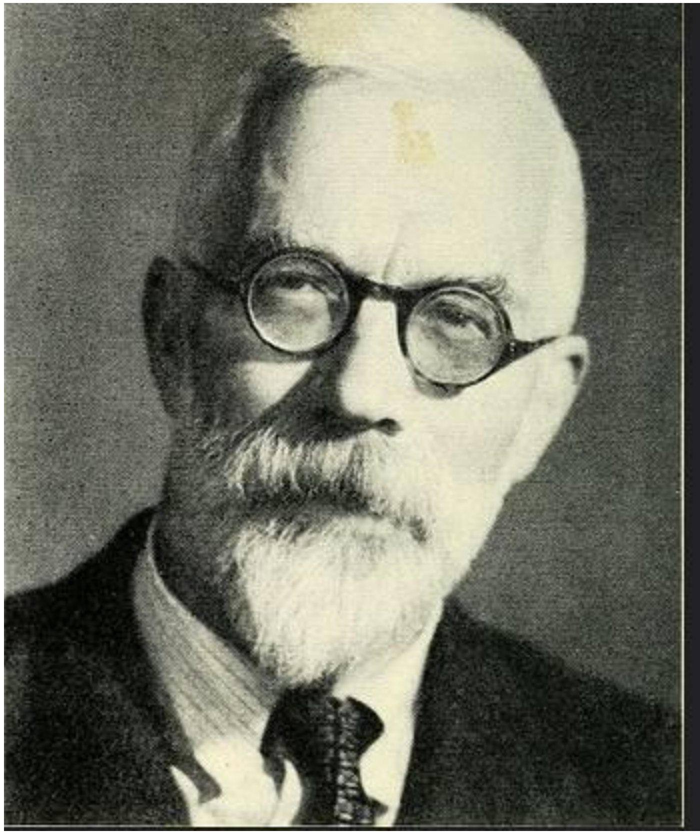
null hypothesis: a statement about the true state of nature

test statistic: T = f(data) indicates the degree to which the data deviate from the null hypothesis

p-value: probability of again observing T or something even more extreme in repeated experiment

Fisher suggests p-value of less than “1 in 20” (0.05) as useful threshold

But was aware of arbitrariness.

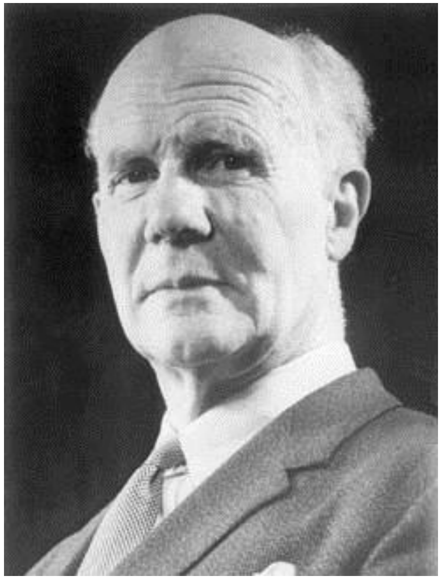


1928 - Jerzy Neyman and Egon Pearson

Is there a best test (for a given problem)?

Say we have two tests for the same problem, is there a way to decide which is better?

One has to add an *alternative hypothesis*. Now we have a *type I error* (reject correct null – false positive) and a *type II error* (accept false null – false negative), both with their corresponding probabilities, α and β.



If two tests have the same α but one has a lower β, it is better!

Usually one studies Power of test = $1 - \beta$

In simple cases it is possible to find best test (Likelihood ratio test, Neyman-Pearson lemma)

Another way to view this testing procedure: it is a decision problem: decide which hypothesis is true

Fisher’s Response?

He hated it!

He considered the whole Neyman-Pearson style of testing to be much more complicated than necessary

First controversy of Statistics!

Their disagreements eventually got very personal: Neyman called some of Fisher's work mathematically “worse than useless”; Fisher called Neyman's approach “childish” and “horrifying [for] intellectual freedom in the west”

(To be fair, their very public dislike for each other was not so much about their differences on hypothesis testing but began with a talk of Neymans before the Royal Statistical Society in 1935 were he claimed (wrongly) that the ANOVA based on latin squares (Fishers invention) was invalid)

Fisher vs Neyman

Fisher was a working Scientist first (Rothamstead experimental station, biology and genetics)

Most important question for him: does my theory agree with my data?

Neyman was a Mathematician (student of Sergei Bernstein), interested in the theory of testing and optimality.

Fisherian Test

- Method: z-test
- $H_0 : \mu = 10$
- $\mathrm{Z} = \sqrt{n}\frac{\overline{X}-\mu}{\sigma} = \sqrt{50}\frac{10.75-10}{3}$ =1.767
- $p = 2 * P(Z > 1.767) = 0.077$
- $0.077 > 0.05$, fail to reject H_0

No alternative, no type I error (but still 5%)

p-value: strength of evidence against null

Neyman-Pearson Test

- Method: z-test
- $\alpha = 0.05$
- $H_0 : \mu = 10$
- $H_a : \mu \neq 10$
- $\mathrm{Z} = \sqrt{n}\frac{\overline{X}-\mu}{\sigma} = \sqrt{50}\frac{10.75-10}{3}$ =1.767
- |Z|<1.96, fail to reject H_0
- Draw power curve

No p-value!

Who won?

Today’s hypothesis testing is a hybrid of both, which neither would have liked.

It is, however much closer to Neyman-Pearson than Fisher.

Fisherian testing survives in Goodness-of-fit tests (H_0: Data comes from normal distribution) because there the space of alternatives is just to large to be useful.

They both agreed on one thing, though:

No Bayesian Statistics! (back then called *inverse probability*)

Fisher: *“The theory of inverse probability is founded up on an error, and must be wholly rejected” (1925, Statistical Methods for Research Workers)*

→ The second controversy in Statistics: Frequentist vs Bayesian Statistics

Thomas Bayes 1701 – 1761

Bayes’ Formula appears in

“*An Essay towards solving a Problem in the Doctrine of Chances*” (176**3**)

Fisher was a great admirer of Bayes, but argued that Bayes was not a Bayesian! (just used uniform prior)

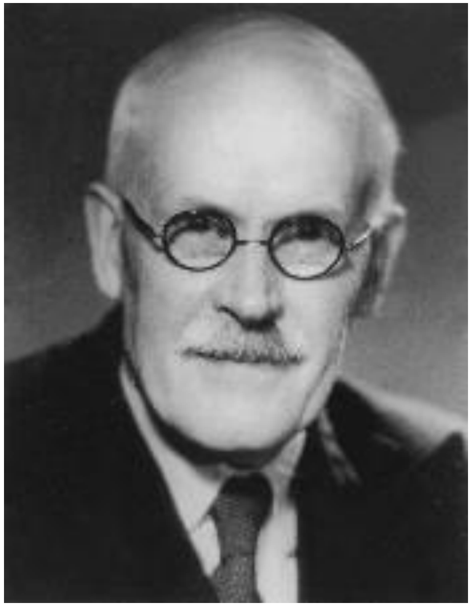
Historians agree that the true father of Bayesian Statistics was Pierre-Simon Laplace (1749-1827)

For most of its history (1900-1990) almost all statistics was Frequentist.

Beginning around 1965 and especially since about 1990 Bayesian statistics has become much more common, even mainstream.

Bruno de Finetti, Jimmy Savage, Harold Jeffreys, Dennis Lindley

Doing anything with Bayesian statistics used be hard, now is much easier (Markov Chain Monte Carlo, Gibbs Sampler etc)



Bayesian Statistics

As in Frequentist Statistics, parameters are viewed as fixed quantities, but our lack of knowledge about them is expressed in terms of probability → priors

Frequentist Statistics: $P(\text{Data}|\text{Hyp})$

Bayesian Statistics: $P(\text{Hyp}|\text{Data})$

They are connected via Bayes' Formula:

$$P(\text{Hyp}|\text{Data}) \sim P(\text{Data}|\text{Hyp})P(\text{Hyp})$$

$P(A|B) \neq P(B|A)$

preg = “Randomly selected person is pregnant”

fem = “Randomly selected person is female”

$$P(\text{preg}|\text{fem}) \approx 3\%$$

$$P(\text{fem}|\text{preg}) \gg 3\%$$

A Bayesian Solution

$H_0: \mu = 10$ vs $H_a: \mu \neq 10$

Need priors $P(H_0)$ and $P(H_a)$

(Say) $P(H_0) = 1/2$

Under $P(H_a)$ $\mu \sim N(10, \tau)$

$$P(H_a|\bar{X} = 10.75) = [1 + \frac{P(H_a)}{P(H_0)} \frac{\exp\{\frac{1}{2}Z^2(1+{}^{\sigma^2}/_{(n\tau^2)})^{-1}\}}{\{1+{}^{n\tau^2}/_{\sigma^2}\}^{\frac{1}{2}}}]^{-1}$$

τ=1 → $P\left(H_a\middle|\bar{X} = 10.75\right) = 0.4 < 0.5$ → accept H_0

τ=3 → $P\left(H_a\middle|\bar{X} = 10.75\right) = 0.6 > 0.5$ → accept H_a

Issues with Bayesian Hypothesis Testing

Intrinsically subjective: $P(H_0)$ and $P(H_a)$

Needs different prior from corresponding estimation problem

Math is much harder

But gives the right answer: P(Hyp|Data)

Is there a problem with NHST?

Lakatos (1978) I wonder whether the function of statistical techniques in the social sciences is not primarily to provide a machinery for producing phony corroborations and thereby a semblance of "scientific progress" where, in fact, there is nothing but an increase in pseudo-intellectual garbage

Rozeboom (1997) NHST is surely the most bone-headedly misguided procedure ever institutionalized in the rote training of science students

So what’s wrong with NHST?

First of all: p-values!

Misinterpretation of p-values as probability that null hypothesis is true

This would be P(Hyp|Data)!!

Only Bayesians can calculate this

Historically in part to blame on confusion of Fisherian and Neyman-Pearson testing

p - value

What’s so special about $p<0.05$?

Absolutely nothing!! But used as a hard cut-off

Lack of understanding that a p-value is a random variable, with its own (often surprisingly large) standard deviation

No real difference between $p=0.04$ and $p=0.06$

It can make a huge difference in real live, though:

p-hacking

Hard to get paper with non-significant result ($p>0.05$) accepted for publication

(This issue is problematic on its own: it ignores the fact that on occasion the fact that nothing was found might actually be interesting. This practice also can lead others to repeat the same research, which they might not have done if they had known about the unsuccessful one. Finally it leads to a bias towards stat. sign results, which makes any meta-analysis hard)

So you worked hard on your research, had a good theory, did an experiment, but got $p=0.1$

All that effort wasted!

But hold on: just do the experiment again, hopefully then $p<0.05$!

Saved! Let’s get that Nobel…

Problem: probability theory says if null is true, p value has uniform distribution, so eventually we are guaranteed to get p<0.05!

→ p hacking

Also known as *torturing the data until it confesses*

A related claim: p-values are not reproducible

So say you did an experiment and got p=0.03, now you repeat the exact same experiment, should you get p=0.03 again?

Of course not, p value is a random variable! And if H_0 is true, or almost so, p has uniform distribution

But many people think they should

Why $\alpha = 0.05$?

Because Fisher said so in his book

But is it reasonable to use the same discovery threshold if

Commit type I error → loose some money

Commit type I error → people die

Or in these cases:

H_0: men and women is USA have the same income (a null which we all are pretty sure is wrong)

H_0: Aliens have never visited planet earth

(a null which most of us would be quite surprised if it turns out to be wrong)

“Extraordinary claims require extraordinary evidence”

The straw-person null

Our null hypothesis: $H_0 \colon \mu = 10$

But in fact we all know that $\mu \neq 10$
After all, $10.0001 \neq 10$

In many sciences null hypotheses are known a-priori to be false – so why test them?

Not all sciences: High Energy Physics 2011

$$H_0 \colon \text{Higgs Boson does not exist}$$

Connected to this:

$$\mathrm{Z} = \sqrt{n}\frac{\overline{X} - \mu}{\sigma}$$

And we reject null if |Z|>crit

Now if null is false $\overline{X} \to \mu_a \neq \mu$ so

$|Z| \to \infty$ as $n \to \infty$

And so we will reject null as long as sample size is large enough, no matter how close the true mean is to the hypothesized one.

Statistical vs Practical Significance

So let’s say it were true that $\mu = 10.1$ for our students.

This would be statistically significant if we tested about 3000 students

But what does it matter?

Such a tiny difference could not possibly have any practical consequences

What has been the response to the BASP ban?

Mostly negative: ban is silly

Misused doesn't mean wrong

Typical Example:

I share the editors' concerns that inferential statistical methods are open to misuse and misinterpretation , but do not feel that a blanket ban on any particular inferential method is the most constructive response.

Peter Diggle – President, Royal Statistical Society

So what is the solution?

Use confidence intervals instead of testing

A Statisticians response: What????

For us hypothesis testing and confidence interval estimation are two sides of the same coin, if you got one you got the other…

But not a great solution either:

Confidence intervals are also routinely misinterpreted:

A 95% CI for the true mean intelligence score of URP undergraduate students is (9.92, 11.58)

Ah, so the probability that the true iq is in (9.92, 11.58) is 0.95!

NO!!!!!!

Again, this is P(Hyp|Data), and only Bayesians are allowed to talk about those (they call theirs credible intervals)

In fact, the new editorial policy of BASP does not allow the use of CI's either (see Question 2):

With the banning of the NHSTP from BASP, what are the implications for authors? The following are anticipated questions and their corresponding answers.

Question 1. *Will manuscripts with p-values be desk rejected automatically?*

Answer to Question 1. No. If manuscripts pass the preliminary inspection, they will be sent out for review. But prior to publication, authors will have to remove all vestiges of the NHSTP (*p*-values, *t*-values, *F*-values, statements about "significant" differences or lack thereof, and so on).

Question 2. *What about other types of inferential statistics such as confidence intervals or Bayesian methods?*

Answer to Question 2. Confidence intervals suffer from an inverse inference problem that is not very different from that suffered by the NHSTP. In the NHSTP, the problem is in traversing the distance from the probability of the finding, given the null hypothesis, to the probability of the null hypothesis, given the finding. Regarding confidence intervals, the problem is that, for example, a 95% confidence interval does not indicate that the parameter of interest has a 95% probability of being within the interval. Rather, it means merely

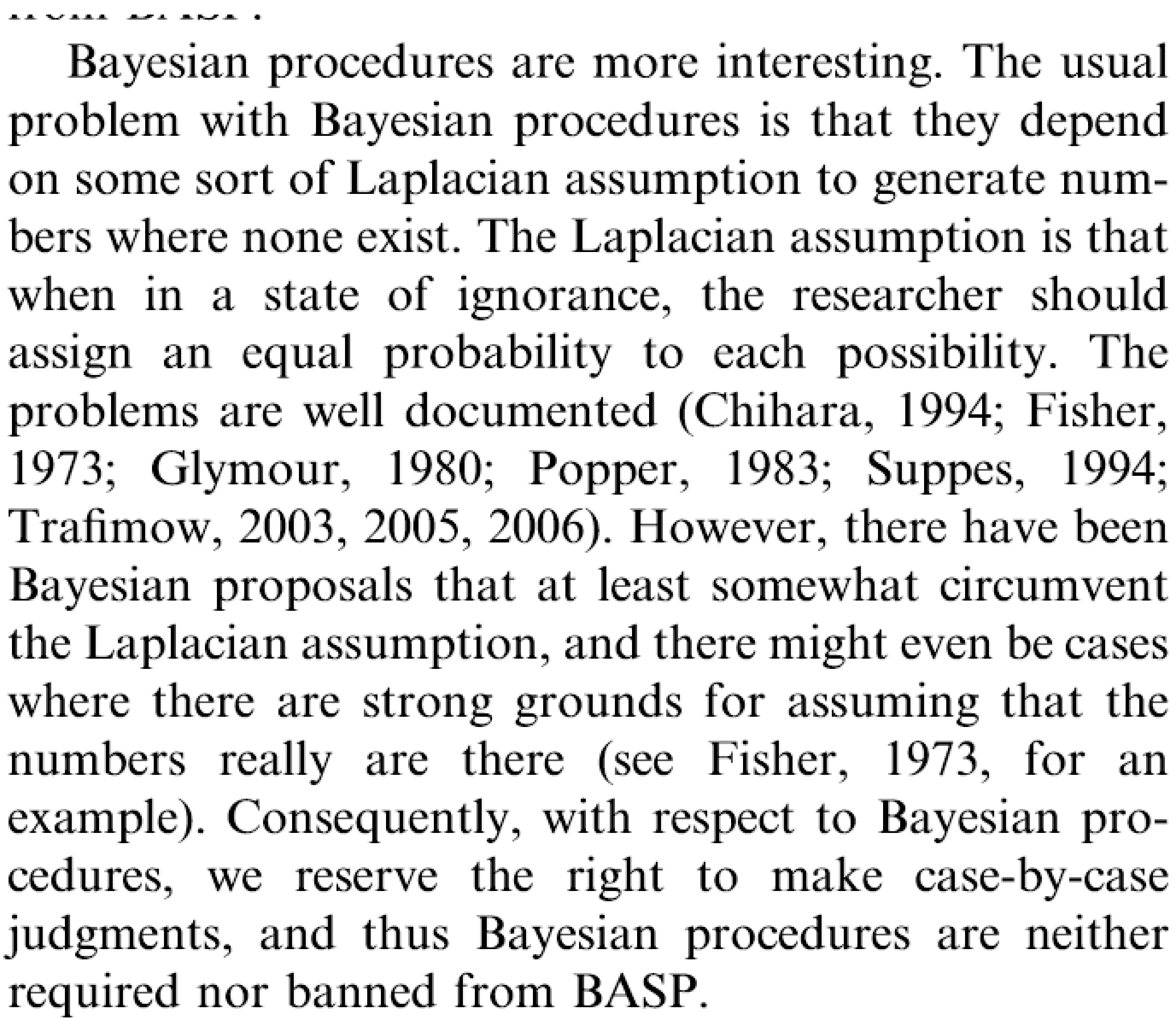
So not even confidence intervals?

This one is fairly new, replacing NHST with confidence intervals used to be the recommendation of most critics of NHST

What’s left? Just giving effect sizes (point estimates)

But as Statisticians know, without standard errors these are useless, and with standard errors we are (almost) back at confidence intervals!

What do they say about Bayesian methods?



Bayesian procedures are more interesting. The usual problem with Bayesian procedures is that they depend on some sort of Laplacian assumption to generate numbers where none exist. The Laplacian assumption is that when in a state of ignorance, the researcher should assign an equal probability to each possibility. The problems are well documented (Chihara, 1994; Fisher, 1973; Glymour, 1980; Popper, 1983; Suppes, 1994; Trafimow, 2003, 2005, 2006). However, there have been Bayesian proposals that at least somewhat circumvent the Laplacian assumption, and there might even be cases where there are strong grounds for assuming that the numbers really are there (see Fisher, 1973, for an example). Consequently, with respect to Bayesian procedures, we reserve the right to make case-by-case judgments, and thus Bayesian procedures are neither required nor banned from BASP.

So, Bayes it is?

Except…

Yes, it does give the right answer, P(Hyp|Data), but

at the price of a prior, and a subjective one at that.

Even a Bayesian test will not solve all the problems:

Will also always reject a false null if sample size is large enough → does not solve straw-person null problem

Also needs an arbitrary cut-off for accepting null or alternative

Is a lot of work even for very simple problems

Needs a lot of mathematical sophistication to be properly understood (and correctly applied)

I will not try to teach the formula for $P(H_a|\bar{X})$ to students who have troubles with calculating a standard deviation!

What I think

In most cases a confidence interval is better than a hypothesis test

As long as everyone is clear on their limitations (P(Data|Hyp) only) Frequentist methods work just fine

There should always be a discussion of what α is appropriate (consequences of wrong decisions) and what effect size is interesting (power of test).

Bayesian methods are always better (if you can agree on a prior, if you understand the math and if you know about things like MCMC, sensitivity analysis etc.)

What I also think

We (statisticians) have to accept some of the guilt for this problem

→ our methods are either to hard to use (Bayesian) or to easy to misunderstand (Frequentist)

→ we are not doing a great job of explaining these things well

Except, a lot of scientists using statistical methods actually were never taught by Statisticians but took home-grown courses in their own departments from their “local expert”

Not How, but Which-What

Traditional introductory Statistics courses focus on How to do something (calculate standard deviation, find a confidence interval etc)

All of these things are better done by computer

We should teach our students:

Which analysis method is appropriate for my experiment?

What does the statistical analysis tell me (and what not?)

We might also teach some basics of the philosophy of Science, such as that theories can only be proven wrong (*falsified*) but never proven right.

But it is hard enough to teach our students the How, is it not impossible to teach concepts like the meaning of a p-value?

Technology to the rescue!

Example: Shiny apps

For our testing problem:

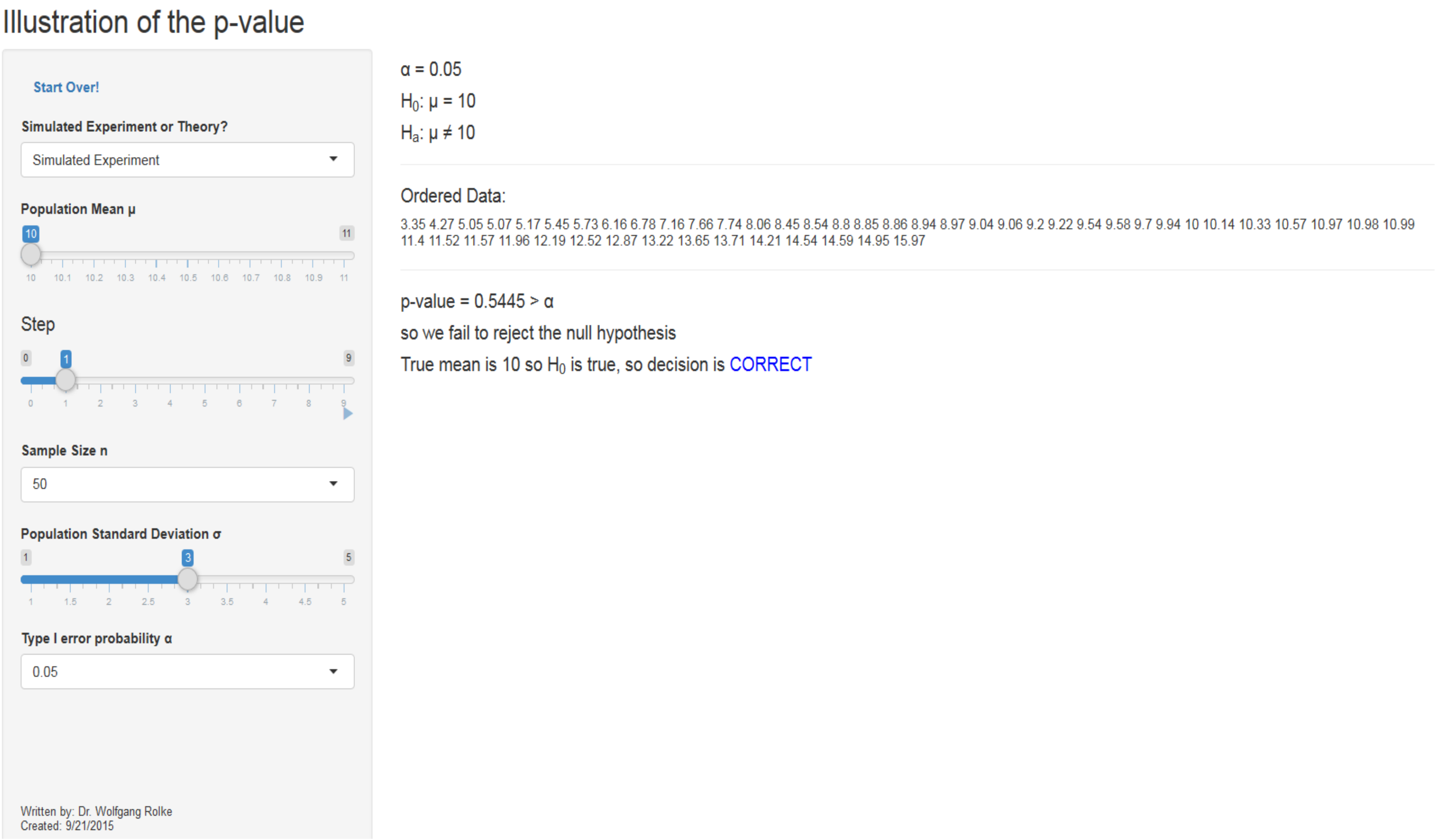


Illustration of the p-value

Simulated Experiment or Theory?

Simulated Experiment

Population Mean μ

10

Step

1

Sample Size n

50

Population Standard Deviation σ

3

Type I error probability α

0.05

Written by: Dr. Wolfgang Rolke
Created: 9/21/2015

$\alpha = 0.05$

H_0: $\mu = 10$

H_a: $\mu \neq 10$

Ordered Data:

3.35 4.27 5.05 5.07 5.17 5.45 5.73 6.16 6.78 7.16 7.66 7.74 8.06 8.45 8.54 8.8 8.85 8.86 8.94 8.97 9.04 9.06 9.2 9.22 9.54 9.58 9.7 9.94 10 10.14 10.33 10.57 10.97 10.98 10.99 11.4 11.52 11.57 11.96 12.19 12.52 12.87 13.22 13.65 13.71 14.21 14.54 14.59 14.95 15.97

p-value = 0.5445 > α

so we fail to reject the null hypothesis

True mean is 10 so H_0 is true, so decision is CORRECT

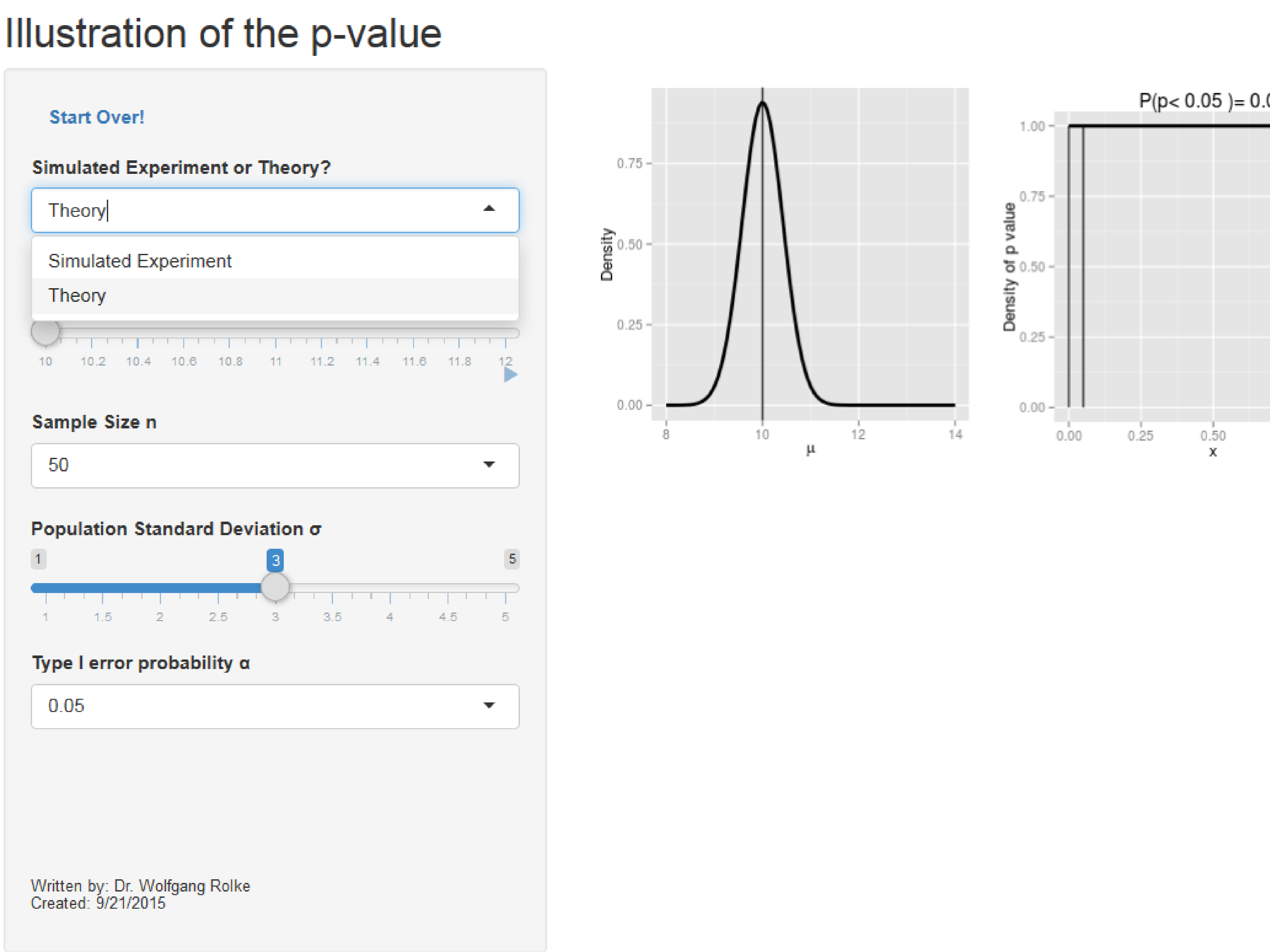


Illustration of the p-value

Start Over!

Simulated Experiment or Theory?

Theory

Sample Size n

50

Population Standard Deviation σ

3

Type I error probability α

0.05

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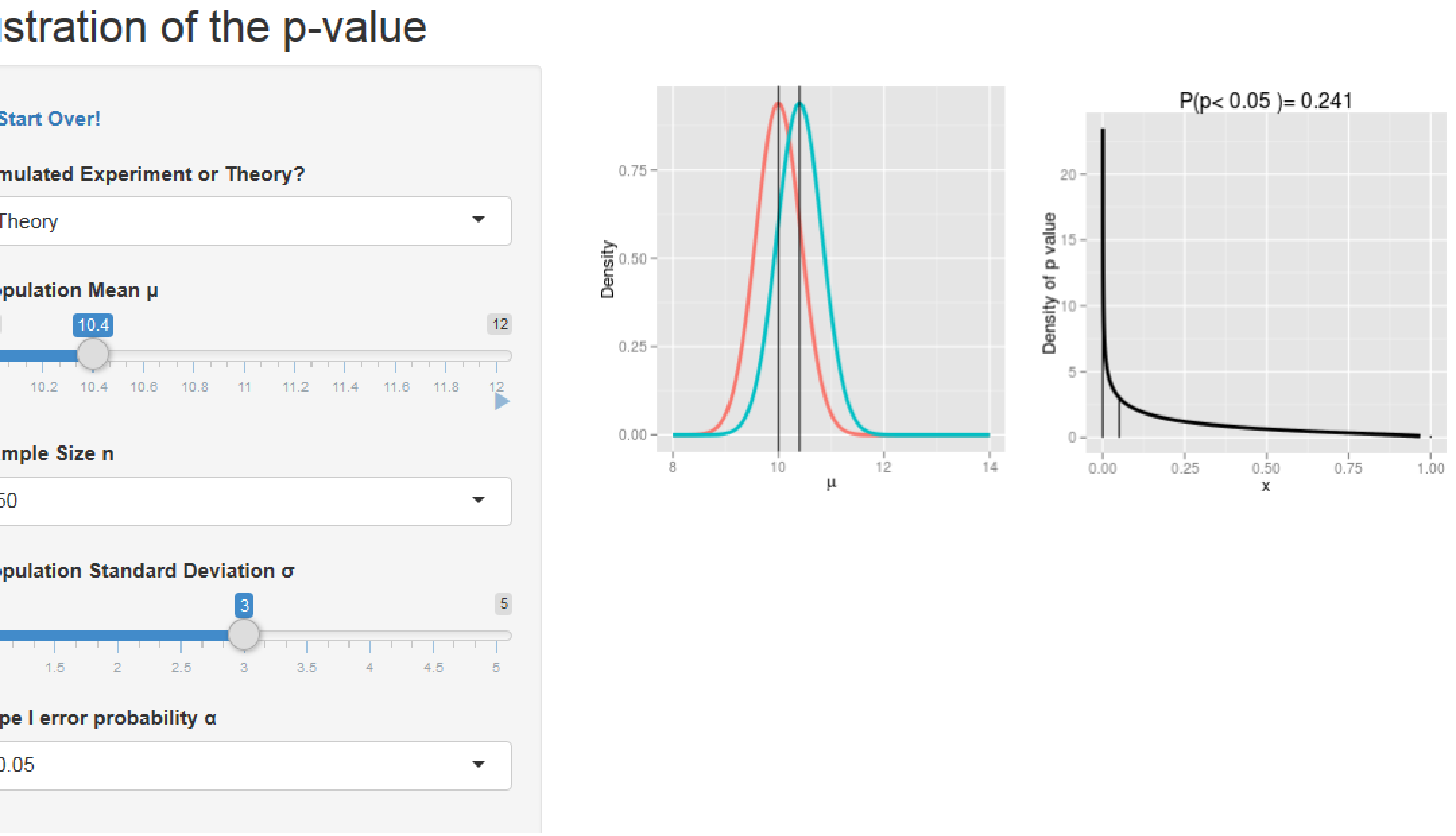

ustration of the p-value

Start Over!

mulated Experiment or Theory?

Theory

pulation Mean μ

10.4

mple Size n

50

pulation Standard Deviation σ

3

pe I error probability α

0.05

Further Reading

This talk is on my homepage at http://academic.uprm.edu/wrolke/research/publications.htm

Many more shiny apps can be found at

http://academic.uprm.edu/wrolke/myapps.htm

James Berger "Could Fisher, Jeffreys and Neyman have Agreed on Hypothesis Testing?" *Statistical Sciences (*2003)

Jacob Cohen "The Earth is Round (p<0.05)" (1994)

Paul Meehl “ Theory-Testing in Psychology and Physics: A Methodological Paradox” *Philosophy of Science* (1967)

Raymond Nickerson “Null Hypothesis Significance Testing: A Review of an Old and Continuing Controversy” *Psychological Methods* (2000)

David Krantz “The Null Hypothesis Testing Controversy in Psychology” *Journal of the American Statistical Association (*1999)

Thanks!