**Work Hardening**

Dislocations interact with each other and assume configurations that restrict the movement of other dislocations. As the dislocation density increases there is an increase in the flow stress of the material. The dislocations can be either “strong” or “weak” obstacles to the movement of other dislocations, depending upon the types of interactions that occurs between moving dislocations. Work hardening or strain hardening can be described as the strengthening of the material by low temperature plastic deformation.

Percent cold work = \[
\%CW = \frac{A_o - A_d}{A_o} \times 100
\]

- Yield Stress
- Work Hardening

Why?

- Cell Structures
- Pattern Formation
Effect of Cold-Working

- Yield strength ($\sigma_{YS}$) increases.
- Tensile strength (TS) increases.
- Ductility (%EL or %AR) decreases.

\[ \sigma_T = K \varepsilon_T^n \]

\[ n \sim 0.5 \text{ (FCC)} \]
\[ n \sim 0.2 \text{ (BCC)} \]
\[ n \sim 0.05 \text{ (HCP)} \]

$n = \text{strain hardening exponent – measures the ability of a metal to harden}$

\[ \tau_{flow} = \tau_0 + k \sqrt{\rho_{disl}} \]

Where $r_{disl}$: dislocation density
DISLOCATION DENSITY

• Dislocation density: total dislocation length per unit volume of material ...
• ... or, the number of dislocations that intersect a unit area of a random section
• The dislocation density typically determines the strength of a material

• Metals (carefully solidified): $10^3 \text{ mm}^{-2}$
• Metals (heavily deformed): $10^9-10^{10} \text{ mm}^{-2}$
• Metals (heat treated): $10^5-10^6 \text{ mm}^{-2}$
• Ceramics: $10^2-10^4 \text{ mm}^{-2}$
• Single crystal silicon for ICs: $0.1-1 \text{ mm}^{-2}$
Fig. 10.3. Collision of dislocations leads to work-hardening.
Critical Resolved Shear Stress vs Dislocation Density

Schmidt’s Law: slip initiates when \[ M \tau_{CRSS} = \sigma_{ys} \]

tensile test single crystal oriented so that different values \((\cos \phi \sin \lambda)_{\text{max}}\) are generated in most-favored slip system.

- Measure \(\sigma_{ys}\) are different. But, \(\sigma_{ys}/m\) are invariant!

CRSS increases with dislocation density: higher YS
Strengthening via strain hardening

**Figure 11.7** Metal deformation during (a) forging, (b) rolling, (c) extrusion, and (d) drawing.
IMPACT OF COLD WORK

• Yield strength increases.
• Tensile strength (TS) increases.
• Ductility (%EL or %AR) decreases (dramatically).

Adapted from Fig. 7.18, *Callister 6e*. (Fig. 7.18 is from *Metals Handbook: Properties and Selection: Iron and Steels*, Vol. 1, 9th ed., B. Bardes (Ed.), American Society for Metals, 1978, p. 221.)
The importance of strain hardening is because in deformation processes one requires high percentage of strain.

Design of a cold-working process

Slab

Material becomes stronger and harder to deform

Material is so hard, backup rolls needed to keep work rolls from deforming
A typical shear stress- shear strain curve for a single crystal shows \textbf{three stages of work hardening}:

- \textit{Stage I} = \textit{“easy glide”} single slip system;
- \textit{Stage II} \textit{with high, constant hardening rate}, multiple slip systems, nearly independent of temperature or strain rate;
- \textit{Stage III} \textit{with decreasing hardening rate, cross slip}, and very sensitive to temperature and strain rate.

BCC systems do not exhibit easy glide.

As the SFE of the FCC alloy decreases, cross slip is more difficult.
Cold-Working Analysis

• What is the tensile strength & ductility after cold working?

\[
\%CW = \frac{\pi r_o^2 - \pi r_d^2}{\pi r_o^2} \times 100 = 35.6\%
\]

\[
\sigma_y \uparrow \text{ from } 150\text{MPa to } 280\text{MPa}
\]

\[
TS \uparrow \text{ from } 220 \text{ to } 320\text{MPa}
\]

\[
\sigma_y \text{ tends to } TS
\]

\[
\delta \downarrow \text{ from } 45\% \text{ to } 7\%
\]
Influence of Cold Working on Mechanical Properties
A re-statement of the governing equation for dislocation strengthening.

\[ \sigma = \langle M \rangle (\tau_0 + \alpha G b \sqrt{\rho}) \quad \text{or} \quad \langle M \rangle (\tau_0 + \alpha G b / \lambda) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>flow stress</td>
<td>Experimentally accessible through mechanical tests</td>
</tr>
<tr>
<td>(&lt;M&gt;)</td>
<td>Average Taylor factor</td>
<td>Magnitude ( \sim 3 ) for tension or compression; depends on the nature of the deformation, the texture and the crystal structure, e.g. (&lt;M&gt;\sim 1.73 ) for torsion (cubic metals)</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>Athermal stress</td>
<td>Contributions from grain size hardening, solutes, etc.</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Geometrical factor</td>
<td>This term accounts for both geometrical factors, and for thermal activation</td>
</tr>
<tr>
<td>( G )</td>
<td>Shear Modulus</td>
<td>Must choose appropriate shear modulus for the slip plane used; Temperature dependent</td>
</tr>
<tr>
<td>( b )</td>
<td>Burgers vector</td>
<td>Derived from the force on a dislocation (Peach-Koehler Eq.)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>dislocation density</td>
<td>Equivalent to the reciprocal of a mean obstacle spacing; depends on work hardening</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>obstacle spacing</td>
<td>Given a number density of particles, the mean spacing, ( \lambda = N^{1/2} )</td>
</tr>
</tbody>
</table>
The effects of plastically deformed polycrystalline metal at temperature less than $T_m$ are:
(a) change in grain shape;
(b) strain hardening;
(c) increased dislocation density and
(d) stored energy.

When metals are plastically deformed about 5% of deformation energy is retained internally associated with dislocations. Most of the applied load that results in plastic deformation is not uniform across the material. Certain amount of stresses is retained in the material even when the load has been eliminated (residual stresses). These stresses are less than the material yield stress. The properties of the cold worked metal (partially or totally) can be restored by:
- Recovery
- Recrystallization and Grain Growth
Recovery

Some of the stored internal strain energy is relieved by virtue of dislocation motion as a result of enhanced atomic diffusion at elevated temperature.

Effects of recovery in cold worked metals:

• Ductility increases
• Yield and tensile strength decreases slightly
• Hardness decreases slightly.
• Metal toughness increases.
• Electrical and thermal conductivity of the metal is recovered to their precold-worked states.
• There is no apparent change in the microstructure of the deformed material.
Recrystallization

Cold worked material
- high dislocation density
- lot of stored energy
- very strong
- not very ductile

Recrystallized material
- low dislocation density
- no stored energy
- weak
- ductile

- After recovery – grains remain at relatively high energy states
- Recrystallization – formation of a new set of strain-free and equiaxed grains, low dislocation densities
- Driving force – difference in internal energy between strained and unstrained material
- New grains form as small nuclei – grow and replace parent material – short –range diffusion
- The process is a heat treating process called annealing. Annealing requires high temperature.
33%CW Brass

Grain Growth after 15 min; and after 10 min at 700°C

4s at 580°C
8s at 580°C
Recrystallization Temperature

Temperature at which recrystallization just reaches completion in one hour. 450°C for the above example.

Typically between 0.5 to 0.33 the melting point of the metal.

Depends on the amount of cold work and of the impurity level of the alloy.

There is a critical degree of cold work below which recrystallization can not be made to occur.
The variation of recrystallization temperature with percent cold work for iron. For deformations less than the critical (about 5% CW), recrystallization will not occur.

### Table 7.2  Recrystallization and Melting Temperatures for Various Metals and Alloys

<table>
<thead>
<tr>
<th>Metal</th>
<th>Recrystallization Temperature</th>
<th>Melting Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>°C</td>
<td>°F</td>
</tr>
<tr>
<td>Lead</td>
<td>-4</td>
<td>25</td>
</tr>
<tr>
<td>Tin</td>
<td>-4</td>
<td>25</td>
</tr>
<tr>
<td>Zinc</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Aluminum (99.999 wt%)</td>
<td>80</td>
<td>176</td>
</tr>
<tr>
<td>Copper (99.999 wt%)</td>
<td>120</td>
<td>250</td>
</tr>
<tr>
<td>Brass (60 Cu–40 Zn)</td>
<td>475</td>
<td>887</td>
</tr>
<tr>
<td>Nickel (99.99 wt%)</td>
<td>370</td>
<td>700</td>
</tr>
<tr>
<td>Iron</td>
<td>450</td>
<td>840</td>
</tr>
</tbody>
</table>
Notes on Recrystallization:

- The amount of cold work controls the initial recrystallized grain size. More cold work $\Rightarrow$ more stored energy $\Rightarrow$ easier nucleation $\Rightarrow$ more nucleation sites $\Rightarrow$ smaller grain size.
- The temperature and time of annealing controls the final grain size, if there is substantial growth after recrystallization. Grain growth requires diffusion, and diffusion is faster at higher temperatures. The time at temperature controls the total amount of diffusion.
- A fine grain size has many benefits beyond strength.
- In general, finer grain sizes are more resistant to fatigue and fracture failures, and have more reproducible and homogeneous mechanical properties.
- Finally, in general, metals with fine grain size are also more easily formed in metalworking operations than metals with coarse grain sizes.
Grain Growth

• Grains continue to grow following recrystallization at elevated temperatures
• Energy is reduced as grains grow in size
• As large grains grow – small grains shrink
• Boundary motion – short-range diffusion of atoms from one side of the boundary to the other.
• At a constant temperature

\[ d^n - d_0^n = Kt \]

\( d_0 \) – initial grain diameter at time (t) = 0; K and n are time independent constants; n is generally \( \sim 2 \).
A method of strengthening one phase by means of another phase finely dispersed in it. The arrangement of the phases is obtained by precipitation from a supersaturated solid solution. Hard precipitates are difficult to shear.

\[ \sigma_y \sim \frac{1}{S} \]
Particle Cutting

For particle, surface areas is created and Stacking Faults For chemically ordered particle, also creates APBs.

Meyers and Chawla, Mechanical Metallurgy
**Particle Hardening: bowing versus cutting**

If the particle is harder than matrix, dislocation can avoid cutting by bowing around particle, or traversing around particle at interphase boundary (a complex process).

*Harder to cut* **stiffer, larger particles.**  
*Easy to bow around stiffer, larger particles.**

Crossover
Breaking Angle: $\phi_c$

$$\tau = \frac{Gb}{L'} \cos\left(\frac{\phi_c}{2}\right)$$

**Figure 5.1**
A dislocation held up by a random array of slip-plane obstacles. The stress required to overcome the obstacles depends on the effective spacing ($L'$) between the obstacles along the dislocation line and the angle ($\phi_c$) to which the dislocation bends before it breaks through them. (After L. M. Brown and R. K. Ham, *Strengthening Mechanisms in Crystals*, ed. A. Kelly and R. B. Nicholson, Wiley, New York, 1971, p. 10.)

**Strong Obstacles: $\phi \sim 0^\circ$**

**Weak Obstacles: $\phi \sim 180^\circ$**
Particle Strengthening

Effects: **Particle size and volume fraction.**

![Graph showing the effects of particle size and volume fraction on strengthening increment. The graph plots the radius of γ' particles against the strengthening increment in MN m^-2. Two curves are shown: one for Ni-12.7 at% Al with a volume fraction f = 0.139, and another for a complex alloy. The graph indicates that increasing the volume fraction f leads to an increase in strength.](image-url)
Strain accommodation for a non-deforming particle by shear loops and bypassing (edge and screws).

Figure 5.31
(a) Shear loops, formed at small plastic strains, around Al₂O₃ particles in a Cu-30% Zn crystal. (b) At larger strains, strain accommodation is provided by an array of prismatic loops. (After P. B. Hirsch and F. J. Humphries, Physics of Strength and Plasticity. ed. A. Arvon. M. I. T. Press. Cambridge, 1969.)
\[ \left( \frac{T}{2} \right) \left( \frac{d\theta}{2} \right) = \tau b \, dl \quad \text{where} \quad T = \frac{Gb^2}{2} \]
SIMULATION: PRECIPITATION STRENGTHENING

- View onto slip plane of Nimonic PE16
- Precipitate volume fraction: 10%

Simulation courtesy of Volker Mohles, Institut für Materialphysik der Universität, Münster, Germany (http://www.uni-muenster.de/physik/MP/mohles/). Used with permission.
Particle Size and Volume Fraction

The interparticle spacing, $\Lambda$, is of little practical use as it is has to measured, more useful are relationships between the mean particle size, $d$, and the volume fraction, $f$, of particles present.

The definition of interparticle spacing in terms of the centre to centre spacing of particles, $L$, is:

$$\Lambda = L - d$$

But, from quantitative metallography:

$$L = \frac{1}{\sqrt{N_a}}$$

Where $N_a$ is the number of particles in unit area.

Also, from quantitative metallography, if there are $N_v$ spherical particles in unit volume:

$$N_a = N_v \cdot d$$

And from the definition of volume fraction, $f$:

$$f = \pi \frac{d^3}{6} N_v$$

$$\frac{4}{3} \pi \left(\frac{d}{2}\right)^3 N_v = f$$
Manipulation of these expressions gives the result:

If $f$ is less than 0.01 (1% volume fraction) then the first term is very large compared to 1 and thus we have:

$$\Lambda = d \left( \sqrt{\frac{\pi}{6f}} - 1 \right)$$

From this expression, knowing the mean particle size and the volume fraction of particles we can calculate the mean interparticle spacing and, therefore, from Orowan's expression the increase in strength.

**Using the information below, estimate the increase in strength of a steel with a dispersion of Fe$_3$C particles with a volume fraction of 1%.** The data you require is

- $G = 80$ GPa.
- Lattice parameter of $\alpha$-Fe = 0.2866 nm.
- $d = 100$ nm.

Answer: 13.7MPa
of the Orowan equation is:

$$\Delta \tau = \alpha \frac{Gb}{\Lambda}$$

where $\Delta \tau$ is the increase in flow or critical resolved shear stress caused by the particles with a spacing, $\Lambda$, $\alpha$ is a constant, value $\sim 0.5$, $b$ the Burgers vector and $G$ is the shear modulus.

**Precipitation Hardening**

- Precipitation hardening - process in which small particles of a new phase precipitate in matrix which harden material by forming impediments to dislocation motion.
- Also known as “Age Hardening” because hardness often increases with time (even at room temperature!)

*Requirements:* System must have:
  - an appreciable maximum solubility of one component in the other (several percent)
  - a solubility limit that rapidly decreases in concentration of major phase
Solvus - solubility limit of B in $\alpha$. Beyond this concentration second phase $\beta$ will form.

can heat it up, without melting, to one phase at high temperature

two phases at low temperature

Solvus - solubility limit of B in $\alpha$. Beyond this concentration second phase $\beta$ will form.
• Precipitation hardening is accomplished with two separate heat treatments

• **Step I : Solution heat treatment**: heating composition $C_0$ to $T_0$ until all B atoms dissolved into a (i.e. b phase is removed) and one phase is obtained. Then the alloy is **quenched (rapidly cooled)** to $T_1$ so as to prevent precipitation of any b phase. A metastable situation occurs but diffusion rates are often too slow at $T_1$ to allow b phase precipitation for long periods of time.

• **Step II : Precipitation heat treatment**: Supersaturated a solid (i.e. has more B in it that it should have) heated to $T_2$ where kinetics allow for **controlled** diffusion of B to form b phase. Forms finely divided b phase. Finally, alloy is cooled to stop precipitation.
Precipitation hardening

Solution heat treatment

Quench

(Excessive Particle growth)

Overaging

Precipitation hardening

Zone forming
Discs of Cu atoms 1 or 2 monolayers thick

Lattice Distortions

Overaging

Strength or hardness

Logarithm of aging time

Solvent (Al) atom

Solute (Cu) atom

θ Phase particle

θ' Phase particle

θ'' Phase particle

Lattice Distortions

No Lattice Distortions
Mechanisms of particle strengthening

1) **Coherency Hardening:** differences in density between the particle and the matrix give rise to elastic stresses in the vicinity of the particle.

2) **Chemical Hardening:** creation of new surface when a particle is sheared increases the area of the interphase boundary, which increases the energy associated with the interface and hence an additional force must be exerted on the dislocation to force it through the particle.

3) **Order Hardening:** passage of a dislocation through an ordered particle, e.g. Ni₃Al in superalloys, results in a disordered lattice and the creation of antiphase boundaries.

4) **Stacking-fault Hardening:** a difference in stacking fault energy between particle and matrix, e.g. Ag in Al, increases flow stress because of the different separation of partial dislocations in the two phases.

5) **Modulus Hardening:** a large difference in elastic modulus results in image forces when a dislocation in the matrix approaches a particle. Consider, e.g., the difference between silver particles (nearly the same shear modulus) and iron particles (much higher shear modulus) in aluminum.
Particle Hardening: coherent, incoherent, and intermediate

Small particles of second phase (1-10%) resist dislocation penetration, especially more so than single solute particles. Degree of strengthening depends on size, volume fraction, shape, nature of boundary (coherency hardening).

interphase boundaries that is
incoherent, which has no coherency strains.

Intermediate interphase boundaries coherency strain are relieved by edge dislocation (not quite 1-1 match)
Coherency hardening

Differences in density between the particle and the matrix give rise to elastic stresses in the vicinity of the particle. This has been analyzed on the basis of the elastic stresses that exist in the matrix adjacent to a particle that has a different lattice parameter than the matrix. Ignoring differences in modulus for now, and setting a parameter, \( \varepsilon \), that approximates a strain to characterize the magnitude of the effect. For

\[
\varepsilon_{coh} = \frac{a_{particle} - a_{matrix}}{a_{matrix}} \quad \tau_{coh} = 7|\varepsilon_{coh}|^{3/2} G \left( \frac{rf}{b} \right)^{1/2}
\]
Particle Hardening: coherent, incoherent, modulus,

Stress fields from coherent solute particle: *coherency hardening*

\[
\varepsilon_{coh} = \frac{a_p - a_m}{a_m}, \quad \tau_{coh} = 7|\varepsilon_{coh}|^{3/2} G \left( \frac{rf}{b} \right)^{1/2}
\]

**Chemical Hardening**

Cutting through a particle with a dislocation displaces one half relative to the other by \(b\), thereby creating new interfacial energy of \(2\pi rb\gamma\), where \(\gamma\) is the interfacial energy between the matrix and the particle. The distance over which this energy has to be created occurs at the entry and exit points and so the characteristic distance is of order \(b\). Thus the force is \(\delta E/\delta x\), or, \(F = 2\pi rb\gamma/2b = \pi r\gamma\)
If the dislocations are straight, we can approximate the spacing between particles as $L = 2r/f$. Dividing the force by $bL$ to find the stress, $\tau = \pi f \gamma/2b$.

A more realistic approach produces the following relationship.

$$\tau = 2G \{\gamma/Gr\}^{3/2}(fr/b)^{1/2}$$

Courtney defines a chemical hardening parameter, $\varepsilon_{\chi\eta} = \gamma/Gr$, related to the interfacial energy, modulus and particle size. This parameter is precisely analogous to the same parameter used, e.g. in APB hardening.
Order Hardening

The hardening depends on the product of the antiphase-boundary energy (APBE) and the area swept by a dislocation in a particle. Thus the increase in flow stress is given by: \( \tau = \pi f \{ \text{APBE} \}/2b \)

In general, low values of the APBE not only predict small increments in hardness, but also the result that the dislocations can move through the particles independently of one another. A more detailed analysis, not presented here, shows a square root dependence on volume fraction, with particle size, \( \tau = 0.7 G \varepsilon^{3/2} \sqrt{(fr/b)} \)

\( \varepsilon_{\text{ord}} = \text{APBE}/Gb \)

Important for Ni-based superalloys
Modulus hardening

The line length in the particle is $2r$ and the change in tension is $(G_{\text{particle}} - G_{\text{matrix}})b^2/2$, assuming the same Burgers vector in matrix and particle. Multiplying the two together and dividing by the distance, i.e. the radius, we get:

$$F = b^2(G_{\text{particle}} - G_{\text{matrix}}) = G_{\text{matrix}} b^2 \varepsilon,$$

where $\varepsilon = (G_{\text{particle}} - G_{\text{matrix}})/G_{\text{matrix}}$, a measure similar to that used in solution hardening.

More realistic estimates of modulus hardening lead to the following formula:

$$\tau = 10^{-2} G \varepsilon^{3/2} \sqrt{fr/b}$$

Think of modulus hardening as being caused by a temporary increase in dislocation line energy while it resides within a particle.
Dislocations cutting particle: *modulus hardening*

As dislocation cuts particle and goes halfway

Max change in $E$:

$$\frac{(G_p - G_m)b^2}{2} (2r) = (G_p - G_m)b^2 r$$

$$\varepsilon_{Gp} = \frac{G_p - G_m}{G_m}$$

$$\tau_{Gp} = 0.2 \varepsilon_{Gp}^{3/2} G \left( \frac{rf}{b} \right)^{1/2}$$
The explanation of age hardening depends on understanding the metastable phases that can appear.

Fig. 5.32  (a) Metastable solvus lines in Al–Cu (schematic). (b) Time for start of precipitation at different temperatures for alloy $X$ in (a).
Concerns:
- Precipitation sometimes occurs at room temperature.
- Overaging - if $\beta$ particles are allowed to grow too large strength of alloy can diminish as particles grow. Why?
Example: Precipitation Hardening & Overaging in Al alloys

Aluminum - low density, high ductility, high reserves (8% of earth crust), easily recycled, high corrosion resistance, high luster, high conductivity, but low strength for a metal. Applications include automotive, aerospace, whenever low weight is desirable. Strengthened by precipitation hardening!

Example - Al(ss) (i.e. α phase) matrix with CuAl₂ (i.e. θ phase) precipitates
Precipitation occurs in two stages

**Supersaturated $\alpha$ solid solution**

**Formation of small, coherent, particles of $\theta''$ phase called Guinier-Preston Zones.**
- Stress field associated with lattice strain impedes dislocations more effectively than $\theta$

**Overaging - $\theta''$ continue to grow and become incoherent with matrix.**
- Phase now becomes $q$ phase
- Lattice strain reduced.
- Dislocations motion not as restricted as in $\theta''$ phase
The most quoted age hardening curve is that for Al-Cu alloys performed in the late 40s. Keep in mind that age hardening was known empirically (Alfred Wilm) as a technologically useful treatment from the early days of aluminum alloys.

Higher Cu contents result in higher maximum hardness because larger volume fractions of precipitate are possible.

Fig. 5.37  Hardness v. time for various Al–Cu alloys at (a) 130 °C (b) 190 °C. (After J.M. Silcock, T.J. Heal and H.K. Hardy, Journal of the Institute of Metals 82 (1953–1954) 239.)
• After solution annealing, Al-Cu alloy is at its softest.
• Immediately after quenching, final shaping and machining are conducted before age hardening begins.
• Age hardening may require heating (artificial aging) although many Al-Cu alloys age harden at room temperature (natural aging).
• Some alloys experience precipitation hardening at room T. These have to be refrigerated to prevent hardening.

Example:
Al 2024 Al-Cu rivets and wing plates for aircraft construction. Solution treated, quenched and refrigerated at suppliers plant, shipped in dry ice. Rivets are driven (deformed) in cold state. Wing sections shaped (deformed) in cold state
**Sample Problem** *(From Dieter, p219 (adapted))*

Question: Al-4%Cu (by wt.) has a yield stress of 600MPa. Estimate the particle size and spacing.

Solution: recognize that this stress relates to age hardening beyond the peak hardness. Therefore use the Orowan bowing stress to estimate the stress. \[ \sigma = <M> \]

\[ \tau_{crss} = <M> \frac{Gb}{\Lambda} \]

\( G=27.6 \text{GPa}; \)

\( b=0.25 \text{nm}; \)

\( <M>=3.1: \) spacing = \( 3.1 \times 27,600 \times 0.25 \times 10^{-9} / 600 = 35.7 \text{ nm} \)

- Now we must estimate the volume fraction of particles for which we use the phase diagram, assuming that we are dealing with the equilibrium phase, \( \theta \), which is **54 w/o Cu**, and the \( \alpha \) in equilibrium with it, **0.5 w/o Cu**.

- Wt. % Al = \( (54-4)/(54-0.5) = 93.5; \) wt. % \( \theta = 4-0.5/(54-0.5)=6.5 \)

- **Volume of \( \alpha = 93.5 \text{gm}/2.7 \text{gm/cm}^3 = 34.6 \text{ cm}^3 \)**

- **Volume of \( \theta = 6.5/ 4.443 \text{ gm/cm}^3 = 1.5 \text{ cm}^3 \)**

- **Volume fraction of \( \alpha = 0.96; \) volume fraction of \( \theta = 0.04. \)**

- Use \( \Lambda=4r(1-f)/3f \): \( r = 3 	imes 0.04 \times 35.7 / 4 / (1-0.04) = 1.12 \text{ nm}. \)
Micrograph of precipitation hardened Al alloy aircraft wing segment. GP zones are typically only a few atoms thick and 25 atoms across.
Natural aging curve for 6061 Al alloy

Yield strength, ksi

Aging time (after quench), hrs (log scale)

- 1 day
- 1 week
- 1 month
- 1 year
• Al-Cu and Al-Cu-Mg alloys
  • Al-Cu model alloy, not used much in practice
  • Al-Cu-Mg “Duralumin” (Al-3.5Cu-0.5Mg-0.5Mn) first widely used age hardening alloy (discovered accidentally in 1906 by Alfred Wilm)
• Widely used for aircraft construction
  • 2618 Al-2.2Cu-1.5Mg-1Ni-1Fe developed for Concorde skin to withstand elevated temperature operation (T=130°C at Mach 2)
• Normally roll clad with pure Al or Al-Zn to protect against corrosion
Strength and ductility during precipitation hardening
(Precipitation hardening characteristics of a 2014 aluminum alloy (0.9%Si, 4.4%Cu, 0.8%Mn, 0.5%Mg)}
## Strengthening of Aluminum

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strength ksi</th>
<th>%elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure annealed Al</td>
<td>2.5</td>
<td>60</td>
</tr>
<tr>
<td>Solid solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strengthened with 1% Mn</td>
<td>6</td>
<td>45</td>
</tr>
<tr>
<td>Highly cold worked pure Al</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Precipitation hardened alloy 7075</td>
<td>80</td>
<td>10</td>
</tr>
</tbody>
</table>
Precipitation Strengthening Application

- Aluminum is strengthened by *ordered metastable Al$_3$Li precipitates* formed by alloying.
total YS: \( \sigma_{YS} = \sigma_{PN} + k \sqrt{c} + k_y \frac{1}{\sqrt{d}} + \alpha G b \rho \)