True Stress (σ_T)

True stress is the stress determined by the instantaneous load acting on the instantaneous cross-sectional area

True stress is related to engineering stress:

Assuming material volume remains constant

$$A_o \ell_o = A \ell$$

$$\sigma_T = \frac{P}{A} = \frac{P}{A} * \frac{A_o}{A_o} = \frac{P}{A_o} * \frac{A_o}{A}$$

$$\frac{A_o}{A} = \frac{\ell}{\ell_o} = \frac{\delta + \ell_o}{\ell_o} = \frac{\delta}{\ell_o} + 1 = (1 + \varepsilon)$$

$$\sigma_T = \frac{P}{A_o}(1 + \varepsilon) = \sigma(1 + \varepsilon)$$

True Strain (ε_T)

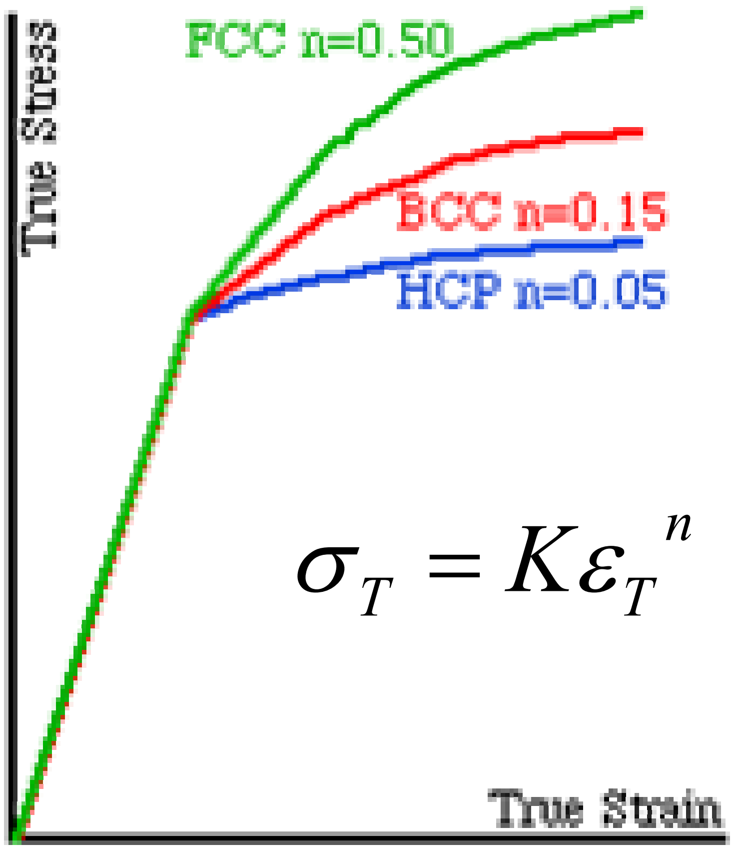
The rate of instantaneous increase in the instantaneous gauge length.

$$\varepsilon_T = \int \frac{d\ell}{\ell} = \ln\left(\frac{\ell}{\ell_o}\right)$$

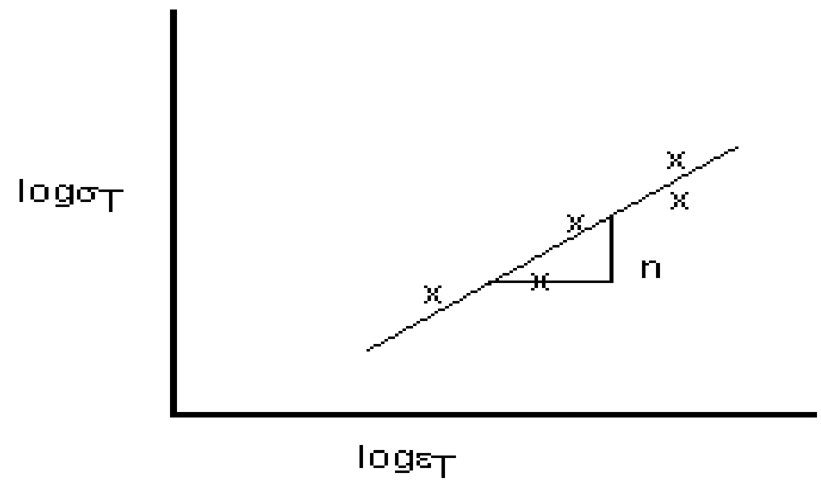
$$\varepsilon_T = \ln\left(\frac{\ell_o + \Delta\ell}{\ell_o}\right) \Rightarrow \ln\left(\frac{\ell_o}{\ell_o} + \frac{\Delta\ell}{\ell_o}\right)$$

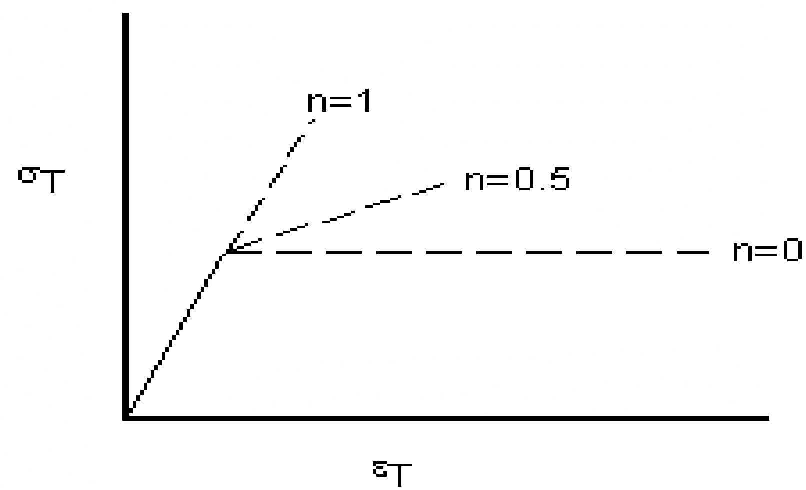
$$\varepsilon_T = \ln(1 + \varepsilon)$$

Strain Hardening Parameter (n)



$$\sigma_T = K\varepsilon_T{}^n$$

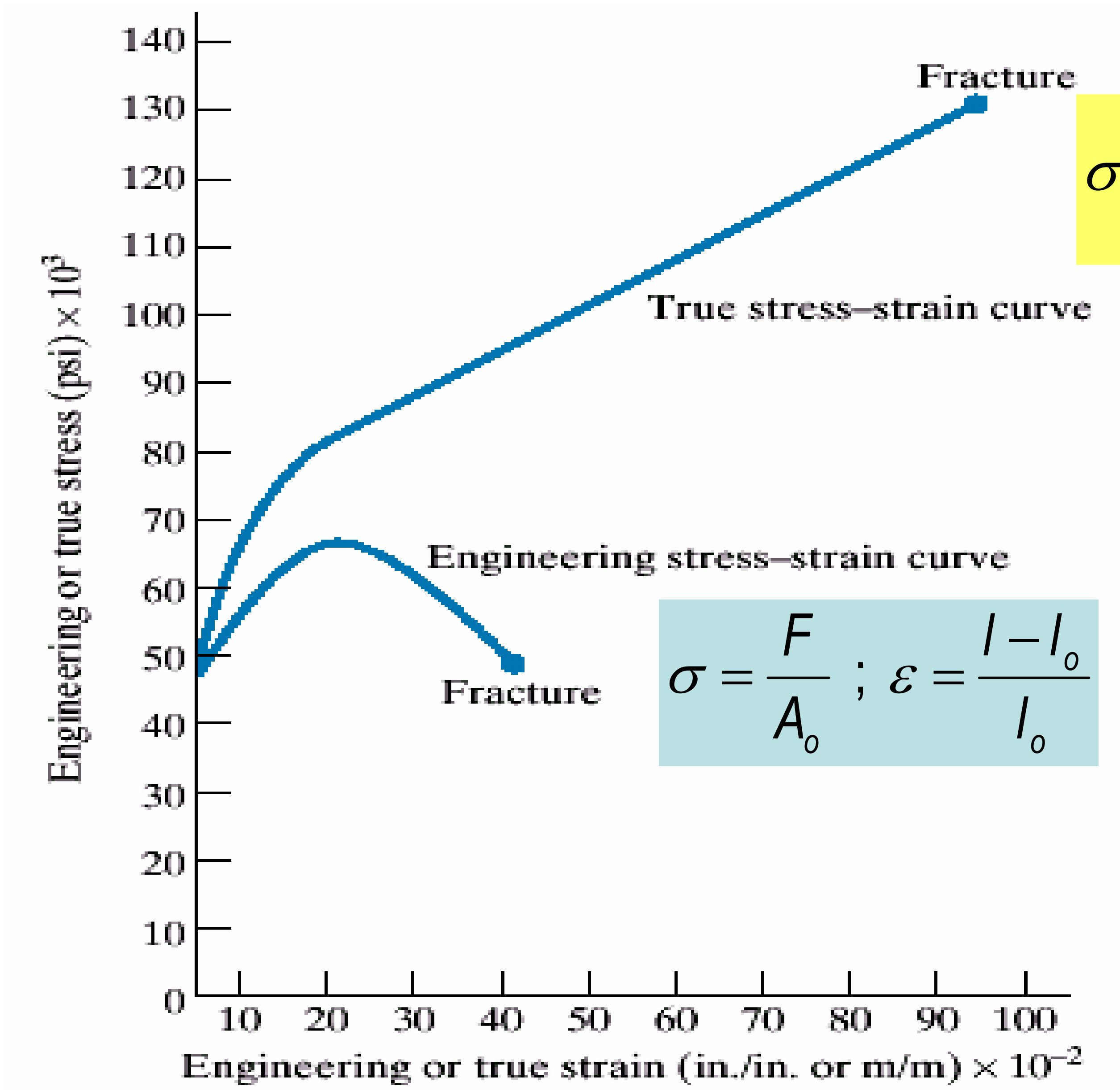




Strain hardening parameter

$$\frac{\sigma_T}{\varepsilon_T} n = \frac{d\sigma_T}{d\varepsilon_T} \qquad 0 \le n \le 1$$

True Stress-Strain Curve



$$\sigma_T = \frac{F}{A_i}\, ; \varepsilon_T = \ln(\frac{l}{l_o})$$

$$\sigma = \frac{F}{A_o}\ ;\ \varepsilon = \frac{l - l_o}{l_o}$$

Table 6.3 Tabulation of *n* and *K* Values (Equation 6.19) for Several Alloys

		K	
Material	*n*	*MPa*	*psi*
Low-carbon steel (annealed)	0.26	530	77,000
Alloy steel (Type 4340, annealed)	0.15	640	93,000
Stainless steel (Type 304, annealed)	0.45	1275	185,000
Aluminum (annealed)	0.20	180	26,000
Aluminum alloy (Type 2024, heat treated)	0.16	690	100,000
Copper (annealed)	0.54	315	46,000
Brass (70Cu–30Zn, annealed)	0.49	895	130,000

Source: From *Manufacturing Processes for Engineering Materials* by Seope Kalpakjian; Copyright © 1984 by Addison-Wesley Publishing Company. Reprinted by permission.

Instability in Tension

Necking or localized deformation begins at maximum load, where the increase in stress due to decrease in the cross-sectional area of the specimen becomes greater than the increase in the load-carrying ability of the metal due to strain hardening.

This conditions of instability leading to localized deformation is defined by the condition $\delta P = 0$.

$$P = \sigma_T A \qquad \delta P = \sigma_T \delta A + A\,\delta\sigma_T = 0 \qquad -\frac{\delta A}{A} = \frac{\delta\sigma_T}{\sigma_T}$$

From the constancy-of-volume relationship,

$$V = A_o l_o = A_i l_i \qquad \frac{\delta l}{l} = -\frac{\delta A}{A} = \delta\varepsilon_T$$

so that at the point of tensile instability

$$\frac{\delta\sigma_T}{\delta\varepsilon_T} = \sigma_T$$

But

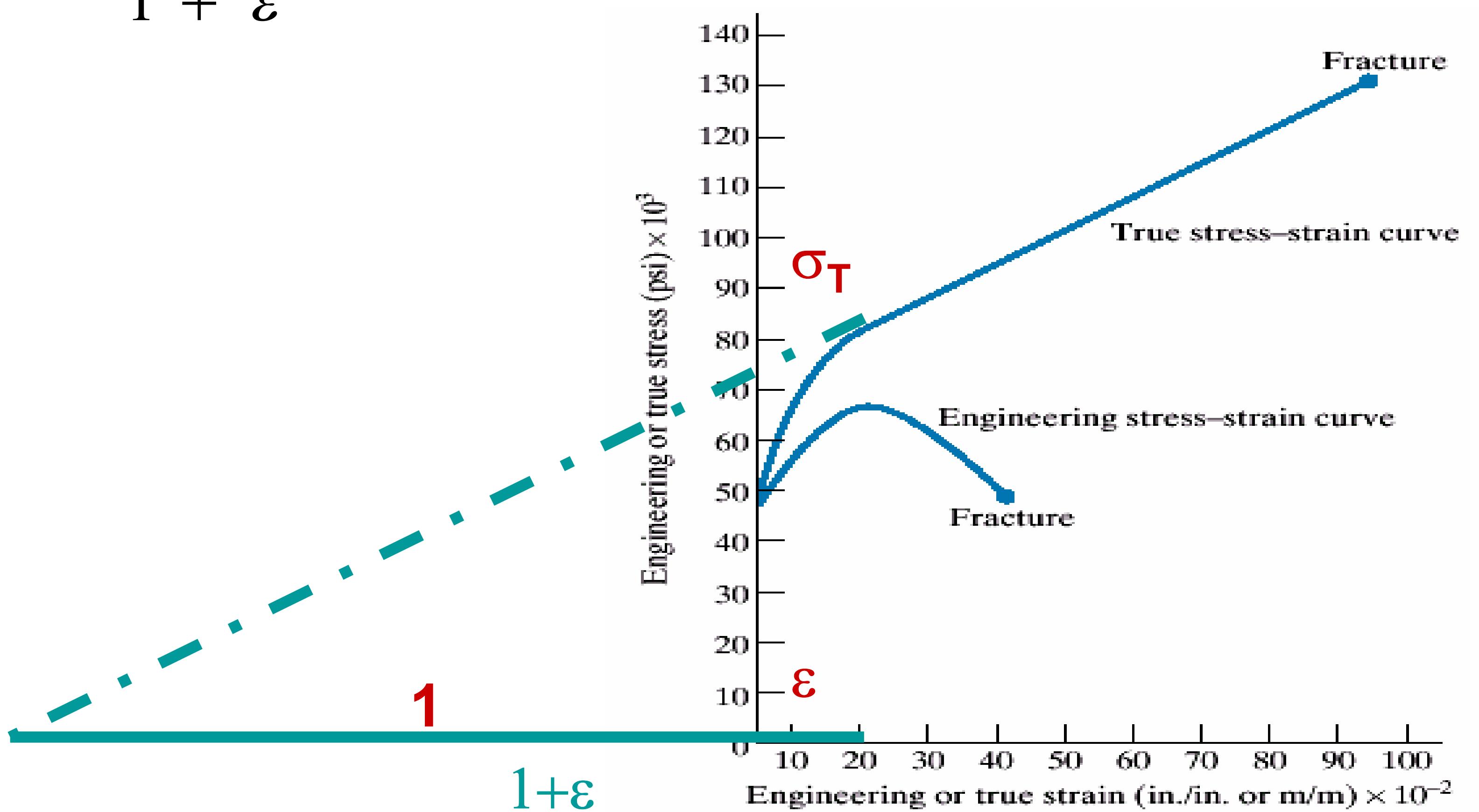
$$\sigma_T = K\varepsilon_T^{\,n} \qquad \frac{\delta\sigma_T}{\delta\varepsilon_T} = Kn\varepsilon_T^{\,n-1} = n\frac{\sigma_T}{\varepsilon_T}$$

Instability occurs when $\varepsilon = n$

The necking criterion can be expressed more explicitly if engineering strain is used.

$$\frac{\delta\sigma_T}{\delta\varepsilon_T}=\frac{\delta\sigma}{\delta\varepsilon}\frac{\delta\varepsilon}{\delta\varepsilon_T}=\frac{\delta\sigma}{\delta\varepsilon}\frac{\delta L/L_o}{\delta L/L}=\frac{\delta\sigma}{\delta\varepsilon}\left(\frac{L}{L_O}\right)=\frac{\delta\sigma}{\delta\varepsilon}(1+\varepsilon)=\sigma_T$$

$$\frac{\delta\sigma}{\delta\varepsilon}=\frac{\sigma_T}{1+\varepsilon}$$



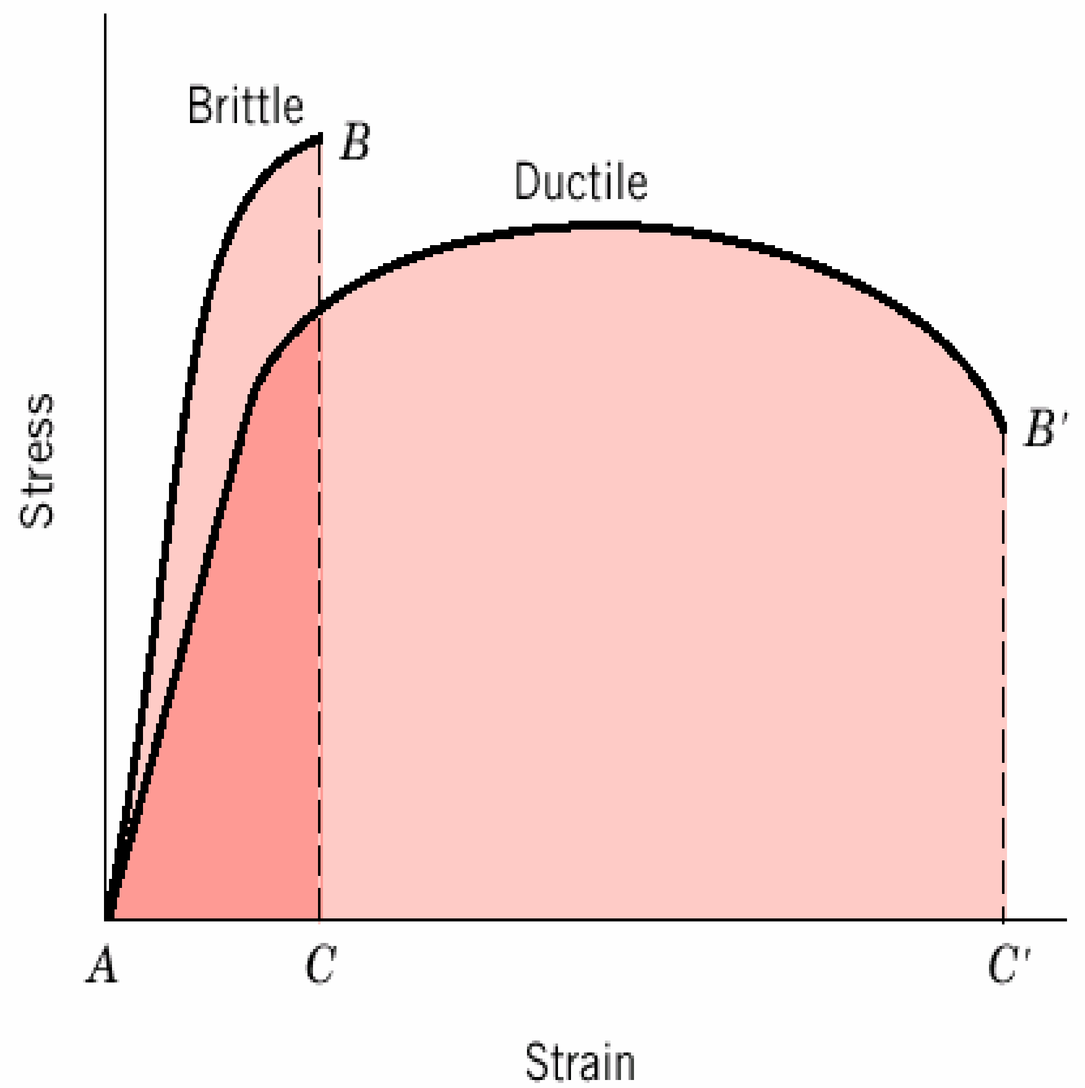
Fracture Behavior

Ductile material – Significant plastic deformation and energy absorption (toughness) before fracture.

Characteristic feature of ductile material - necking

Brittle material – Little plastic deformation or energy absorption before fracture.

Characteristic feature of brittle materials – fracture surface perpendicular to the stress.



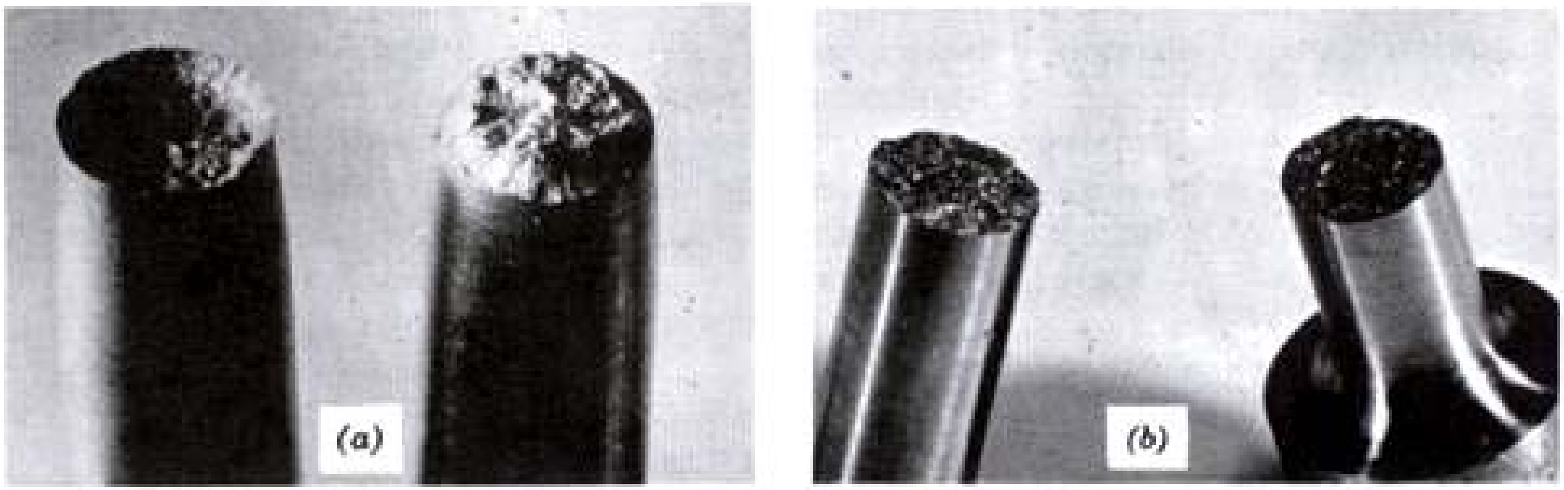
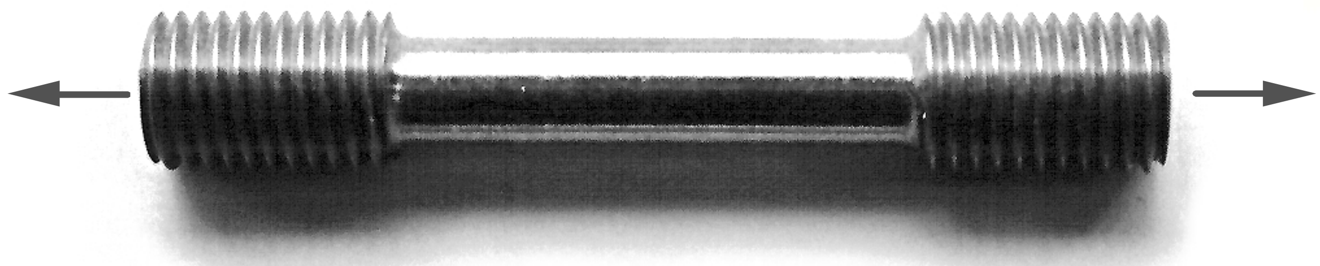
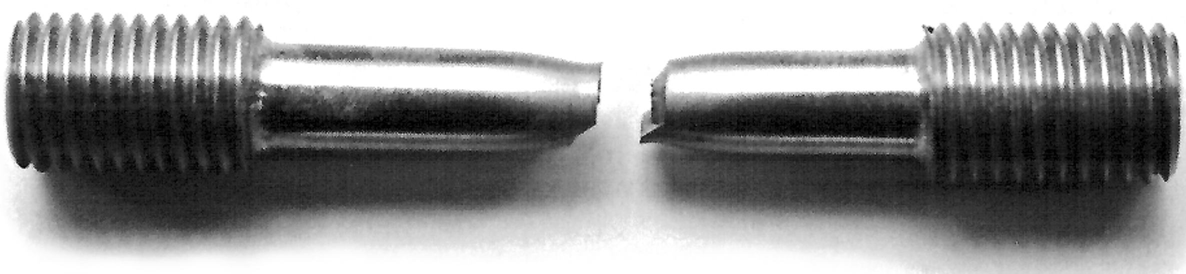


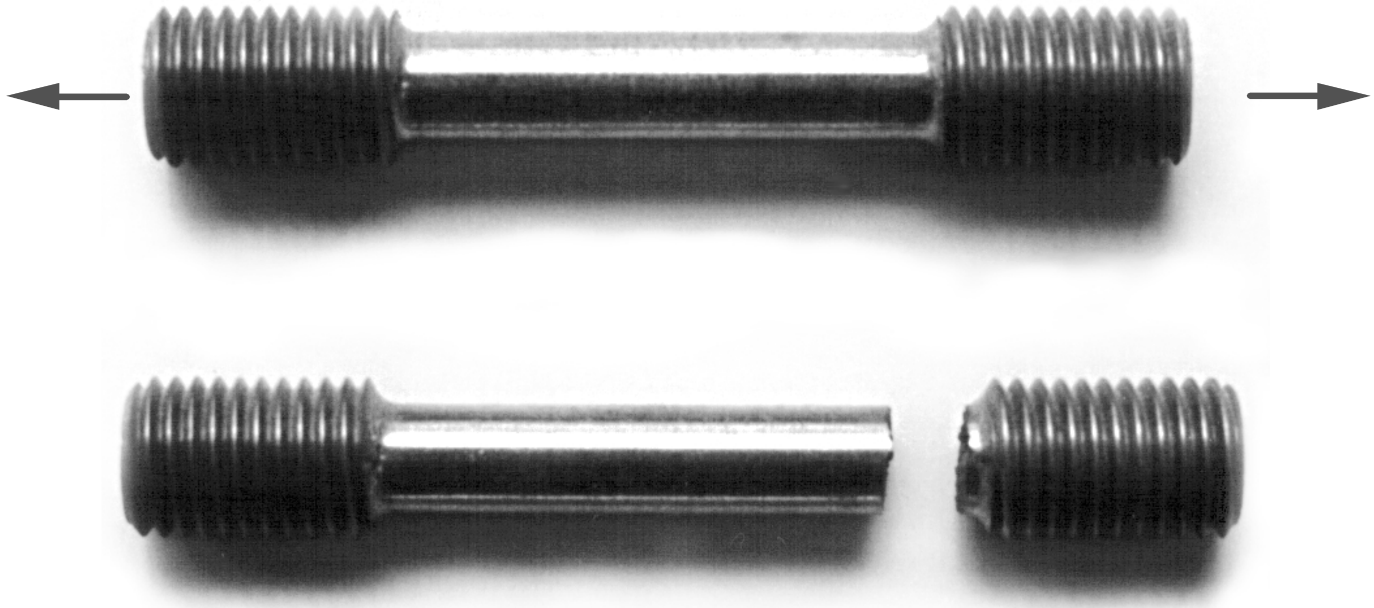
Figure 8.3 *(a)* Cup-and-cone fracture in aluminum. *(b)* Brittle fracture in a mild steel. (From H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 144. Copyright © 1965 by John Wiley &



Steel

Before and after fracture





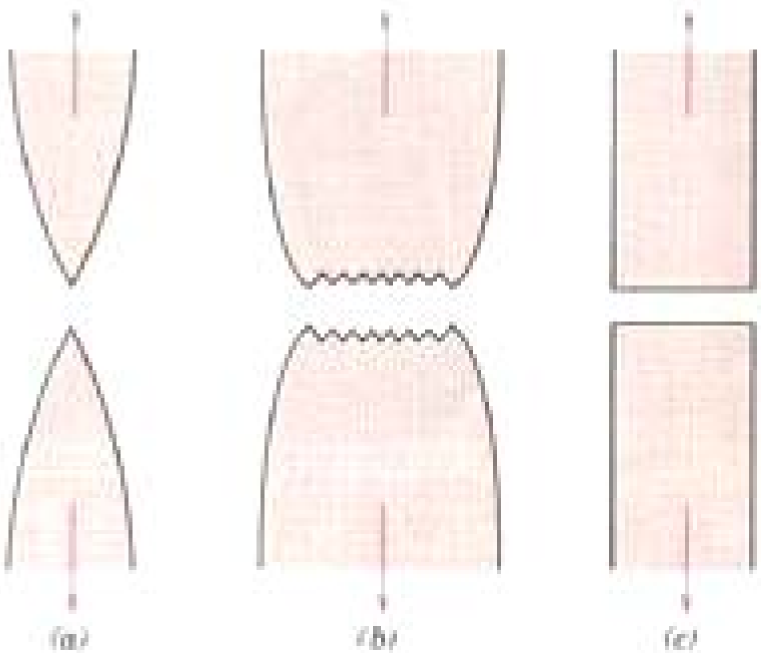
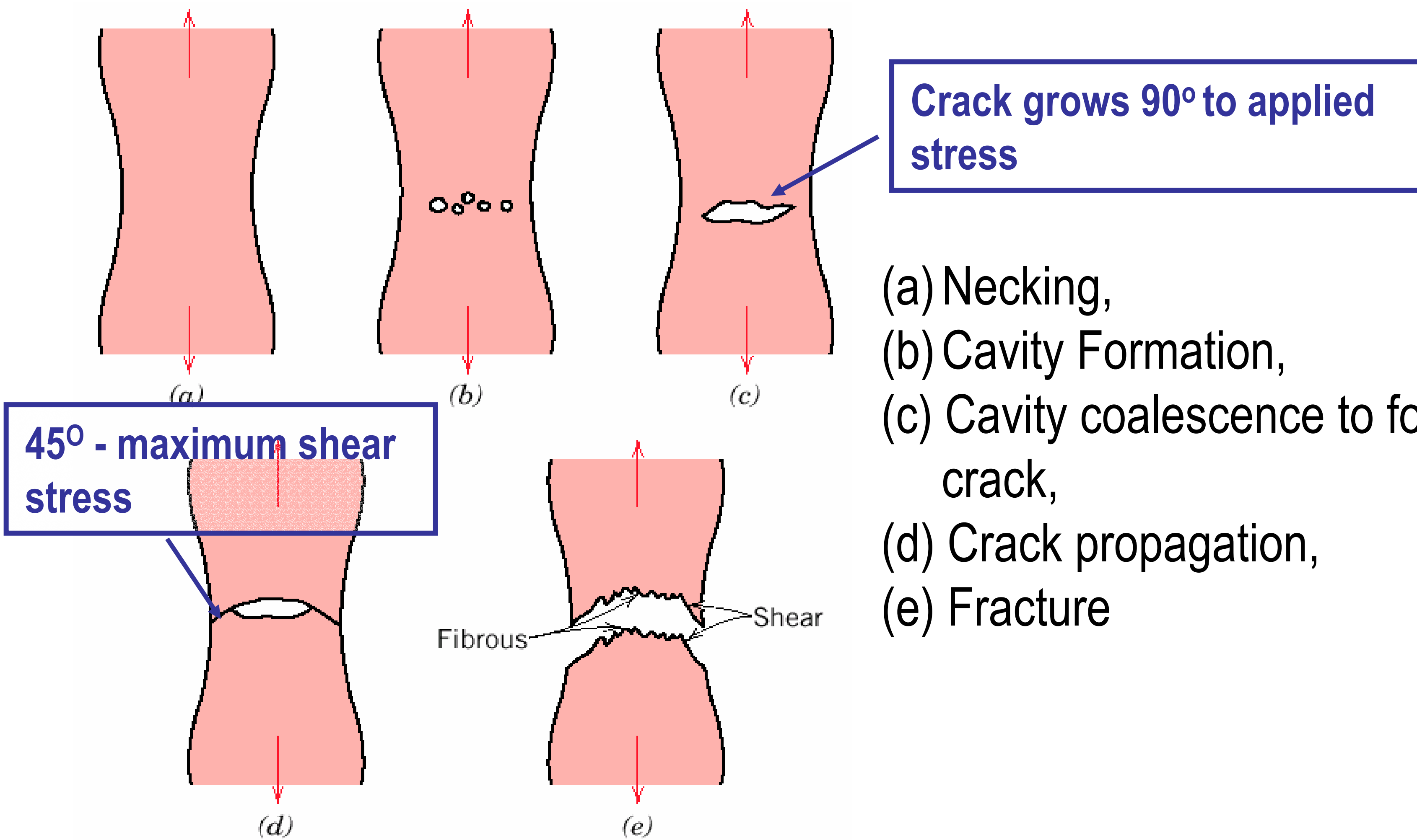
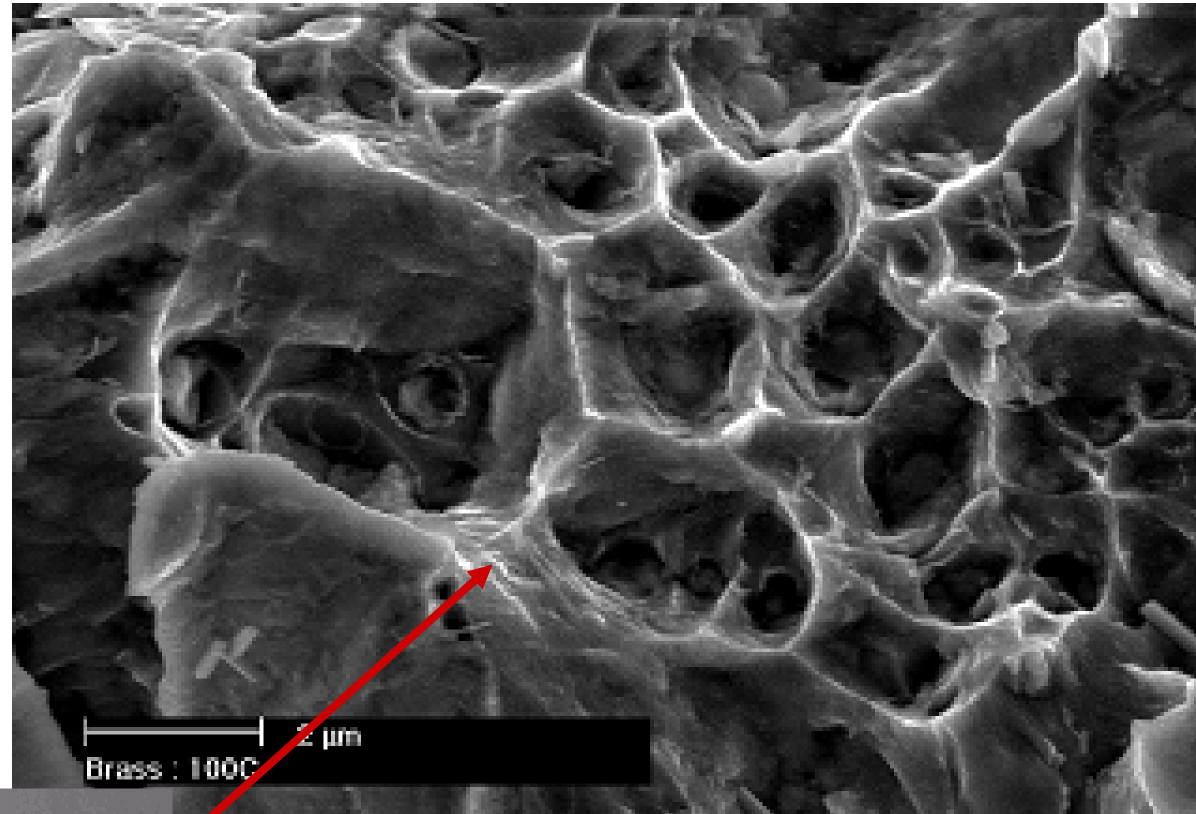


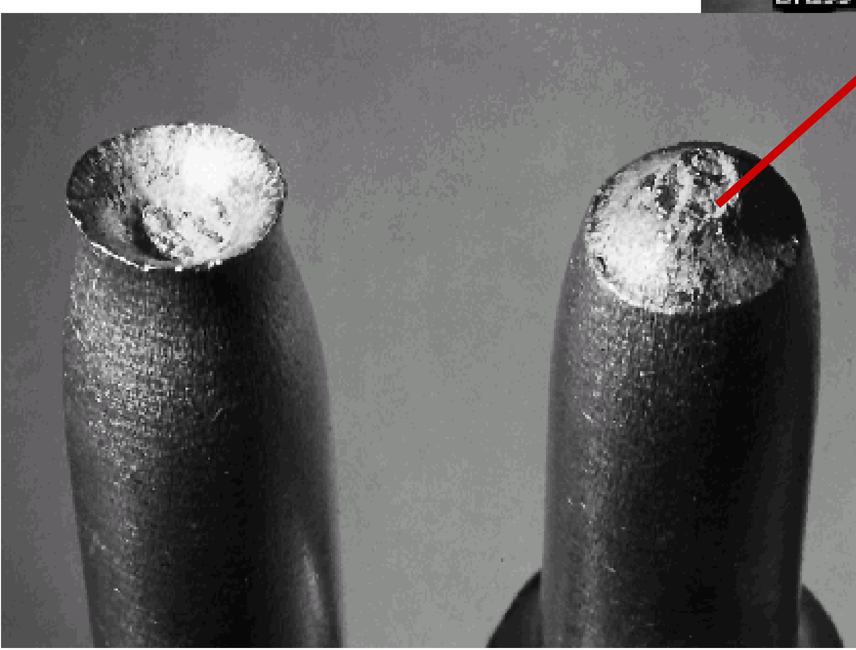
Figure 8.1 (*a*) Highly ductile fracture in which the specimen necks down to a point. (*b*) Moderately ductile fracture after some necking. (*c*) Brittle fracture without any plastic deformation.

Ductile Fracture (Dislocation Mediated): Extensive plastic deformation. Necking, formation of small cavities, enlargement of cavities, formation of cup-and-cone. Typical fibrous structure with "**dimples**".



(a) Necking,
(b) Cavity Formation,
(c) Cavity coalescence to form a crack,
(d) Crack propagation,
(e) Fracture

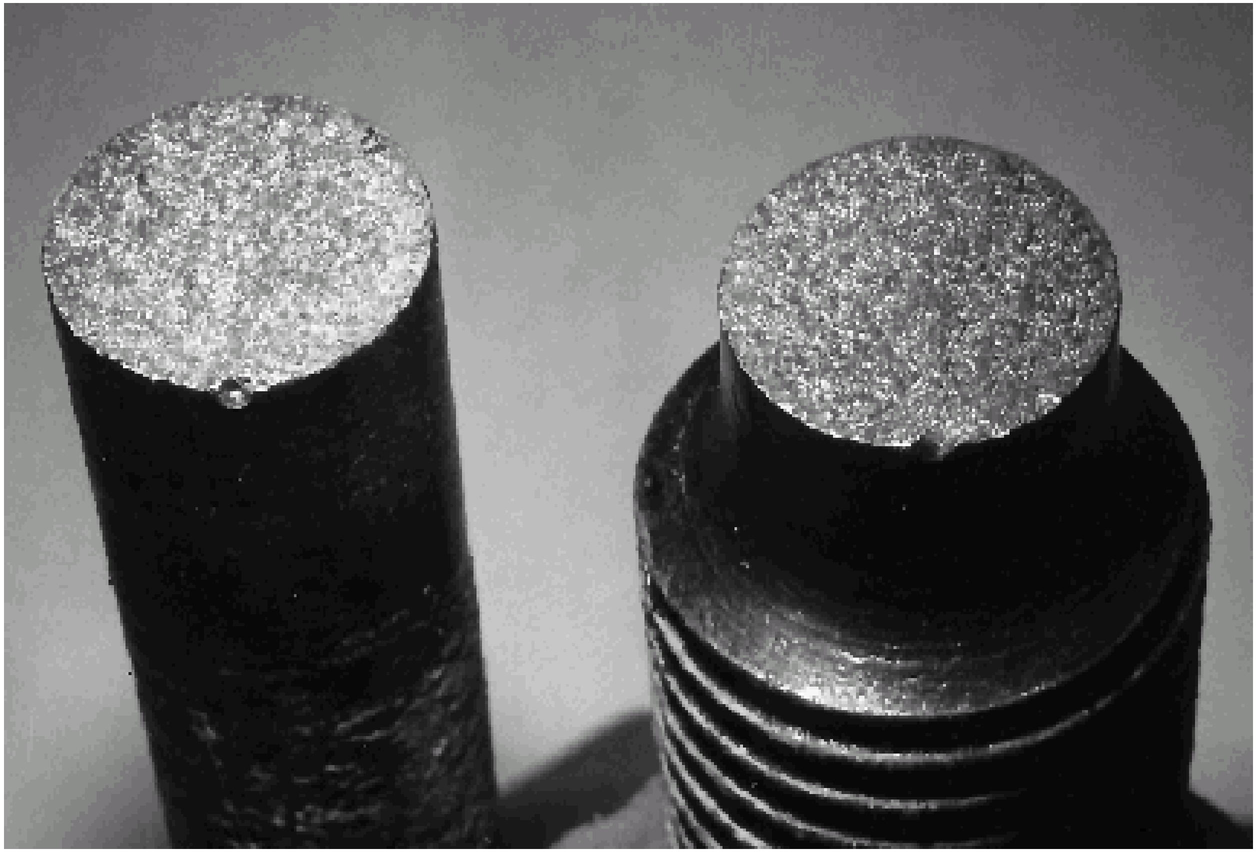


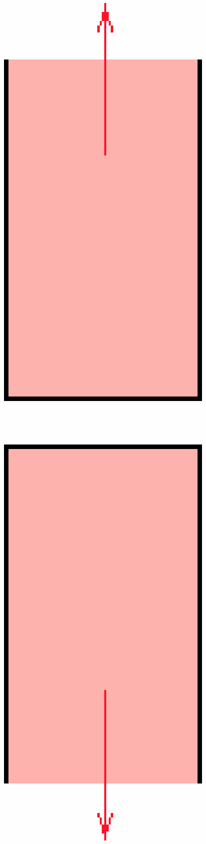


Scanning Electron Microscopy:
Fractographic studies at high resolution.
Spherical “dimples” correspond to micro-cavities that initiate crack formation.

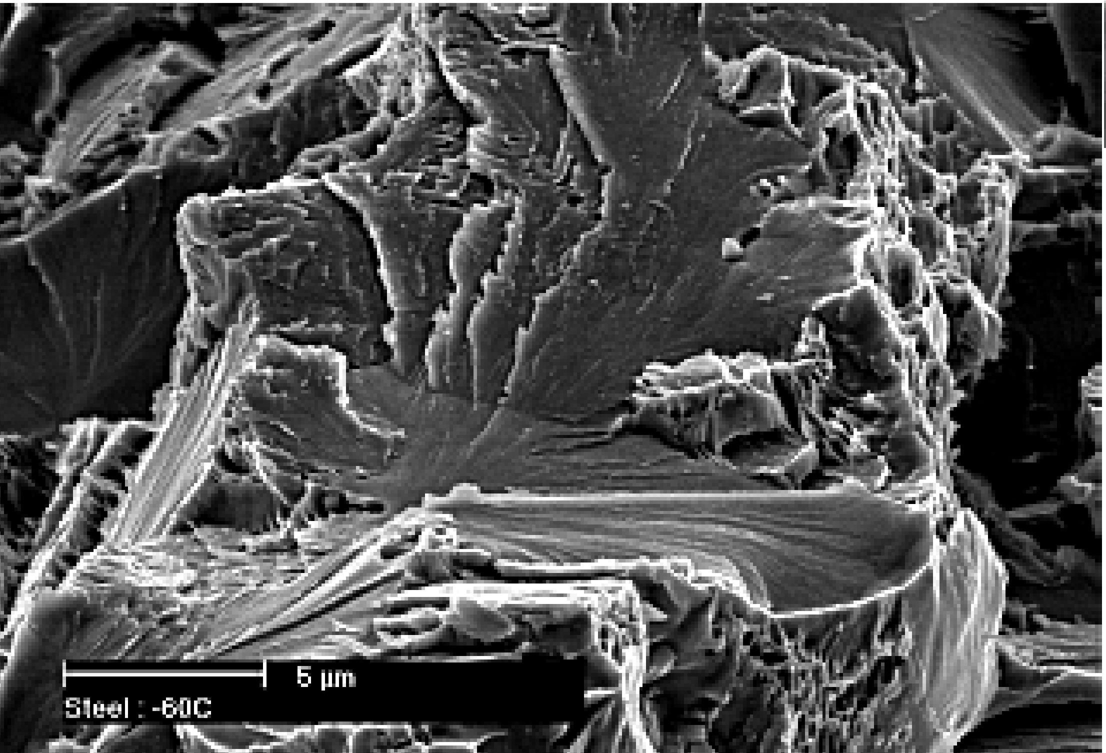
Brittle Fracture (Limited Dislocation Mobility): very little deformation, rapid crack propagation. Direction of crack propagation perpendicular to applied load. **Crack often propagates by cleavage - breaking of atomic bonds along specific crystallographic planes (cleavage planes).**

Brittle fracture in a mild steel





Intergranular fracture: Crack propagation is along grain boundaries (grain boundaries are weakened or embrittled by impurities segregation etc.)



Transgranular fracture: Cracks pass through grains. Fracture surface has faceted texture because of different orientation of cleavage planes in grains.

Stress-Strain Behavior of Ceramics

Flexural Strength: the stress at fracture under the bending tests. It's also called **Modulus of rupture**, **fracture strength**, or the **bend strength**

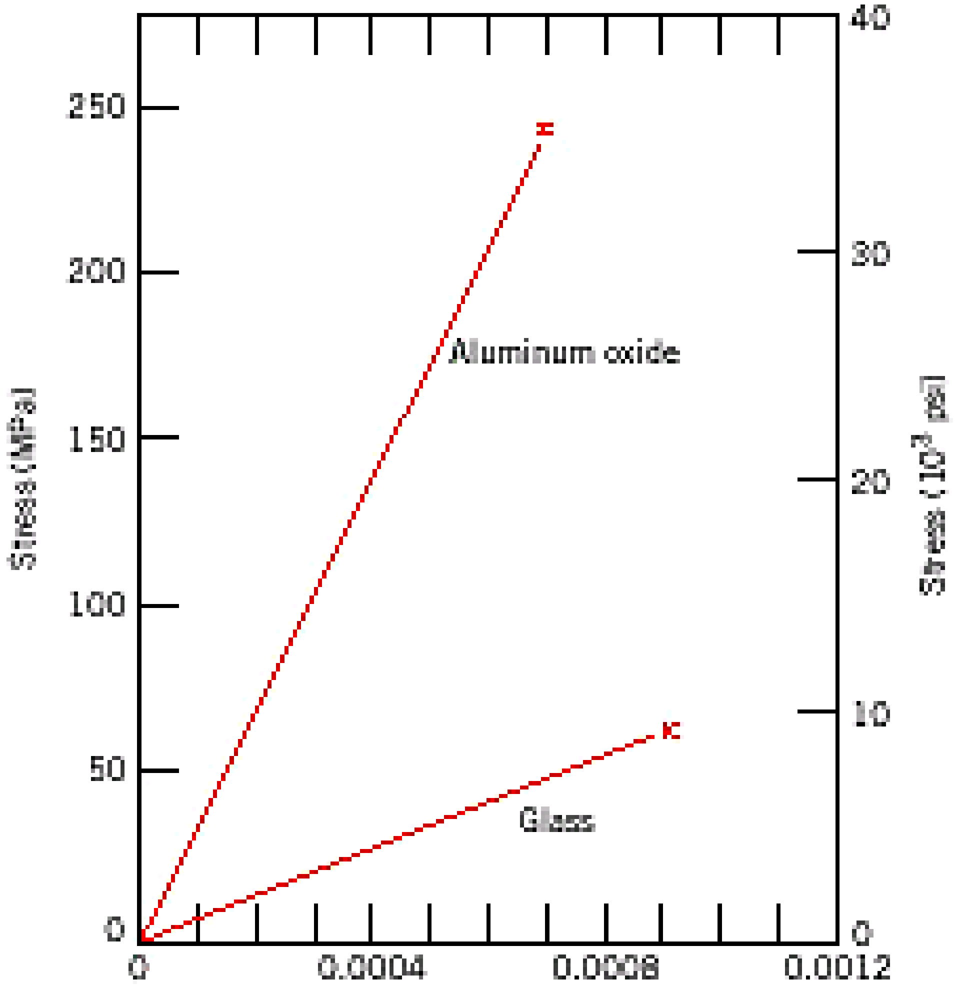


FIGURE 13.29 Typical stress–strain behavior to fracture for aluminum oxide and glass.

3-point Bending tests

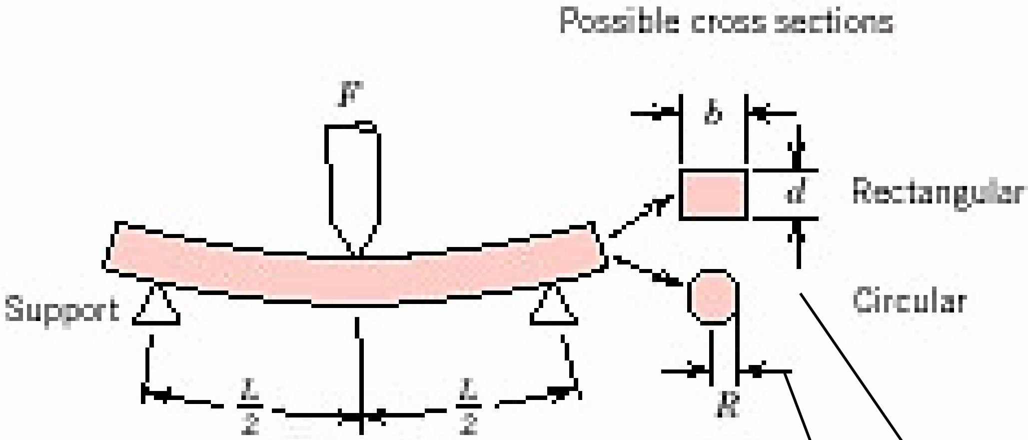


FIGURE 13.28 A three-point loading scheme for measuring the stress–strain behavior and flexural strength of brittle ceramics, including expressions for computing stress for rectangular and circular cross sections.

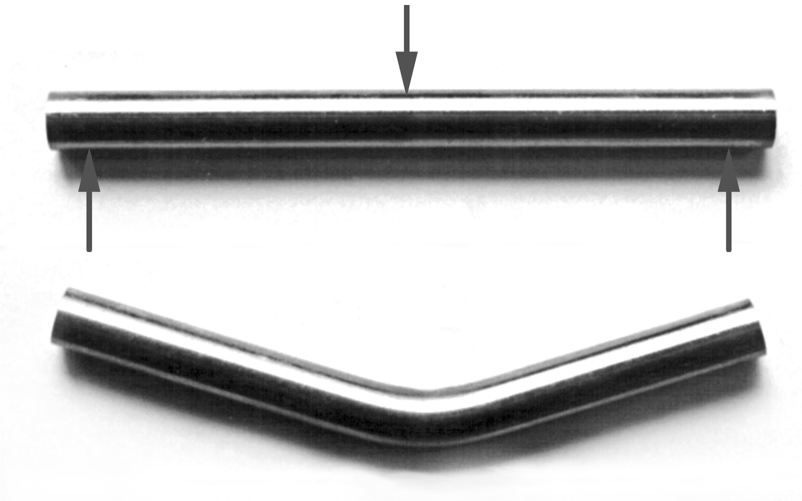
σ = stress = $\frac{Mc}{I}$

where M = maximum bending moment
c = distance from center of specimen to outer fibers
I = moment of inertia of cross section
F = applied load

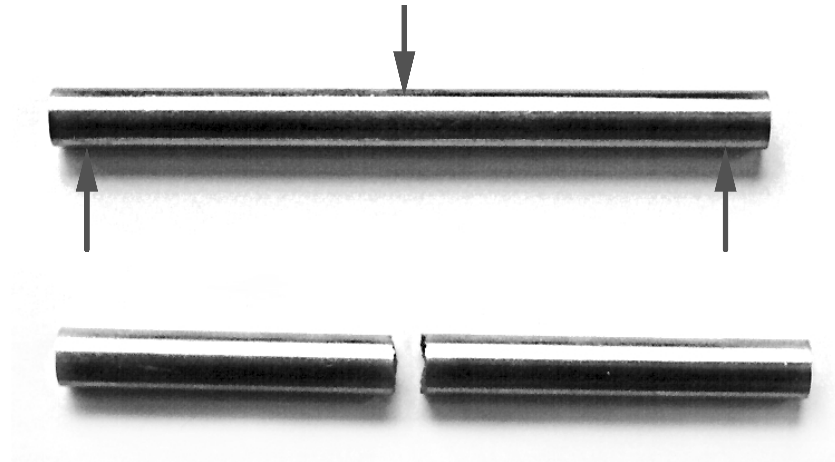
	M	c	I	σ
Rectangular	$\frac{FL}{4}$	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{3FL}{2bd^2}$
Circular	$\frac{FL}{4}$	R	$\frac{\pi R^4}{4}$	$\frac{FL}{\pi R^3}$

$$\sigma_{fs} = \frac{3F_f L}{2bd^2}$$

$$\sigma_{fs} = \frac{F_f L}{\pi R^3}$$



(a)



(b)

Ductile

Brittle