Superconductivity

A material that exhibits zero electrical resistance under certain conditions and expels a magnetic field completely (perfect diamagnetic).

It is also known as the Meissner or Meissner-Ochsenfeld effect

\[ \text{Critical Temperature (T}_C\text{): temperature at which the resistivity of the material plunges from a finite value to zero.} \]

The origin of superconductivity is related to electron-phonon coupling and the resultant pairing of conduction electrons (Bardeen-Cooper-Schrifer or BCS Theory).
Superconductivity disappears when:

\[ T > T_c, \quad H, B > H_c, B_c \quad J > J_c \]

Temperature higher than the critical temperature. Certain critical level of magnetic field is reached. Certain critical current density is reached. The critical magnetic field and current density depend on the temperature.

**Figure 20.22** Critical temperature, current density, and magnetic field boundary separating superconducting and normal conducting states (schematic).
On the basis of the magnetic response, superconducting materials may be classified in two:

**Type I**: materials that in the superconducting state are completely diamagnetic (Meissner effect) and as the magnetic field is increased, the material remains diamagnetic until $H_c$ is reached.

**Type II**: materials that in the superconducting state are completely diamagnetic (Meissner effect) and remain diamagnetic at low magnetic fields, however, the transition from the superconducting state to the normal state is gradual and occurs between the lower critical ($H_{c1}$) and upper critical ($H_{c2}$) fields.

**The Meissner effect**: Observed when a superconductor is cooled below its critical temperature and surface currents appear whose effect is to expel the magnetic field from the interior of the superconductor.

The magnetic field is repelled from the superconductor, this explains levitation.
### Examples of Type I Superconductors and their critical temperatures

<table>
<thead>
<tr>
<th>Element</th>
<th>Critical Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead (Pb)</td>
<td>15.00 K</td>
</tr>
<tr>
<td>Lanthanum (La)</td>
<td>7.196 K</td>
</tr>
<tr>
<td>Tantalum (Ta)</td>
<td>4.88 K</td>
</tr>
<tr>
<td>Mercury (Hg)</td>
<td>4.47 K</td>
</tr>
<tr>
<td>Tin (Sn)</td>
<td>4.15 K</td>
</tr>
<tr>
<td>Indium (In)</td>
<td>3.72 K</td>
</tr>
<tr>
<td>Thallium (Tl)</td>
<td>3.41 K</td>
</tr>
<tr>
<td>Rhenium (Re)</td>
<td>2.38 K</td>
</tr>
<tr>
<td>Protactinium (Pa)</td>
<td>1.697 K</td>
</tr>
<tr>
<td>Thorium (Th)</td>
<td>1.40 K</td>
</tr>
<tr>
<td>Aluminum (Al)</td>
<td>1.38 K</td>
</tr>
<tr>
<td>Gallium (Ga)</td>
<td>1.175 K</td>
</tr>
<tr>
<td>Molybdenum (Mo)</td>
<td>1.083 K</td>
</tr>
<tr>
<td>Zinc (Zn)</td>
<td>0.915 K</td>
</tr>
</tbody>
</table>

### Examples of Type II Superconductors and their critical temperatures

<table>
<thead>
<tr>
<th>Compound and Alloys</th>
<th>Critical Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb-Zr</td>
<td>10.8 K</td>
</tr>
<tr>
<td>Nb-Ti</td>
<td>10.2 K</td>
</tr>
<tr>
<td>Nb₃Sn</td>
<td>18.3 K</td>
</tr>
<tr>
<td>V₃Ga</td>
<td>16.5 K</td>
</tr>
<tr>
<td>Nb₃Ge</td>
<td>23.0 K</td>
</tr>
<tr>
<td>MgB₂</td>
<td>39 K</td>
</tr>
</tbody>
</table>
Ceramic Compounds

<table>
<thead>
<tr>
<th>Compound</th>
<th>$T_c$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{YBa}_2\text{Cu}<em>3\text{O}</em>{7-x}$</td>
<td>(Y-123)</td>
</tr>
<tr>
<td>$\text{Bi}_2\text{Sr}_2\text{CaCu}<em>2\text{O}</em>{8+x}$</td>
<td>(Bi-2212)</td>
</tr>
<tr>
<td>$(\text{Bi,Pb})_2\text{Sr}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>{10+x}$</td>
<td>(Bi-2223)</td>
</tr>
<tr>
<td>$\text{TlBa}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>{9+x}$</td>
<td>(Tl-1223)</td>
</tr>
<tr>
<td>$\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>{10+x}$</td>
<td>(Tl-2223)</td>
</tr>
<tr>
<td>$\text{HgBa}_2\text{Ca}_2\text{Cu}<em>3\text{O}</em>{8+x}$</td>
<td>(Hg-1223)</td>
</tr>
</tbody>
</table>

$\text{Hg}_{0.8}\text{Tl}_{0.2}\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{8.33}$ 138 K (Highest Recorded at Ambient Temp.)
Applications

Magnetic Resonance Imaging (MRI)

The development of superconductors has improved the field of MRI as the superconducting magnet can be smaller and more efficient than an equivalent conventional magnet.

Radiofrequency receivers that are currently made of copper coils can be replaced by superconducting receiver coils, increasing signal-to-noise ratio by a factor of two in some cases. This change is especially important in low-strength MRI fields (based on lower cost magnets), where weak fields mean weak signals. Superconducting coils could boost the performance of these machines by improving image quality and reducing measurement time.
'MagLev' trains have been under development in Japan for the past two decades. Superconducting magnets are used to create a strong magnetic force to propel the vehicle. But they offer more than just propulsion—they also levitate the vehicles and guide them within the bounds of the guideway.

The system takes advantage of the naturally stabilizing effect provided by electromagnet induction. No controlling devices whatsoever are needed to keep the train on its guideway, and there is no risk of the train ‘derailing.’ The magnetic levitation force is ideal for supporting a train at very high speeds.
SuperComputers

Without superconductor: large heat, large electric power use
with superconductor: no heat, small electric power use

Faster (petaflop) computers.
A petaflop is a measure of a computer's processing speed and can be expressed as:
A thousand trillion floating point operations per second (FLOPS)
Today's fastest computing operations have only reached "teraflop" speeds - trillions of operations per second.
Currently the fastest are NEC's Earth Simulator, which operates at a top-end of 40 teraflops
Josephson junction

A thin layer of insulating material sandwiched between 2 superconducting layers. Electrons "tunnel" through this non-superconducting region in what is known as the "Josephson effect". Here a superconducting current flows even in the absence of an external voltage. (Named for Brian Josephson.)
Magnetic Field Sensing – Josephson device

SQUID (Superconducting QUantum Interference Device)

A superconducting loop interrupted in 2 places by Josephson junctions. When sufficient electrical current is conducted across the squid body, a voltage is generated proportional to the strength of any nearby magnetic field.

Compared with the rest of the body’s low current operations, the heart is a highly electric organ. Its faint magnetic field (about 100 picoteslas) can be measured with superconducting quantum interference devices (SQUIDs), the most sensitive magnetic sensors known. When arranged in arrays, SQUIDs can provide an image of the heart’s magnetic field and yield clues to abnormal conduction patterns that are the basis of some heart arrhythmias (abnormal rhythms). About 3 million persons are treated for arrhythmias each year in the United States.
• Electric generators made with superconducting wire are far more efficient than conventional generators wound with copper wire.
  – Their efficiency is above 99% and their size about half that of conventional generators.
  – They make lucrative ventures for power utilities.
• Recently, power utilities have begun to use superconductor-based transformers and "fault limiters".
  – Superconducting fault limiters can respond within a few milliseconds to limit thousands of amperes of current – such as after a lightning strike.
• An idealized application for superconductors is to employ them in the transmission of commercial power to cities.
  – BUT, due to the high cost and impracticality of cooling miles of superconducting wire to cryogenic temperatures, this has only happened with short "test runs".
• Superconducting material takes up less space.
  – In one instance 250 pounds of superconducting wire replaced 18,000 pounds of vintage copper cable, making it over 7000% more space-efficient.
• Among emerging technologies are a stabilizing momentum wheel (gyroscope) for earth-orbiting satellites that employs the "flux-pinning" properties of imperfect superconductors to reduce friction to near zero.

• Ultra-sensitive, ultra-fast, superconducting light detectors are being adapted to telescopes due to their ability to detect a single photon of light.

• Superconductors may play a role in Internet communications soon.
  - In late February, 2000, Irvine Sensors Corporation received a $1 million contract to research and develop a superconducting digital router for high-speed data communications.
  - Since Internet data traffic is doubling every 100 days, superconductor technology is being called upon to meet this super need.
Magnetism

Magnetism, the phenomenon by which materials assert an attractive or repulsive force or influence on other materials.

**Basic Concepts**

An electrical current in a loop generates a magnetic field.

Magnetic fields are generated by moving electrically charged particles.

Earth North Pole is actually the South magnetic pole.

Field lines – come out from the north towards the south

Magnetic field strength - \( H \)
Magnetization, Permeability, and the Magnetic Field

A current passing through a coil sets up a magnetic field (H). Where \( n = \text{number of turns}; \ L = \text{length of the coil and} \ l = \text{current.} \)

Units for H (A/m) and oersted

When H is applied in vacuum, lines of magnetic flux are inducted. The number of lines of flux, called flux density or inductance B, is related to the applied magnetic filed by the equation

\[
B_o = \mu_o H \quad \text{(weber/m}^2\text{)}
\]

\(\mu_o\): magnetic permeability of vacuum.

Units:

H(oersted), B(gauss), \(\mu_o\)(gauss/oersted)

H(A/m), B(weber/m\(^2\) – tesla), \(\mu_o\)(4\(\pi\)x10\(^{-7}\) weber/A.m – henry/m)
When we place a material within the magnetic field, $B$ is determined by the manner in which induced and permanent dipoles interact.

$$B = \mu H \quad \text{(weber/m}^2)$$

$\mu > \mu_0$ magnetic dipole moments reinforce the field

$\mu < \mu_0$ magnetic dipole moments oppose the field

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0}$$

$\mu_r = \text{relative permeability}$

“How much a particular material can be magnetized compared to a vacuum”
Magnetization of a Solid

\[ B = \mu_0 H + \mu_0 M \]

\( \mu_0 \) = permeability under vacuum

\( M \) = magnetization of the material

Review of Terms

\( B \) Magnetic Induction (Tesla or kg/A-s\(^2\) or Wb/m\(^2\))

\( H \) Magnetic field (amp-turn/m or C/m-s)

\( M \) Magnetization (same as magnetic field)

\( \mu_0 \) Permeability (henry/m or kg-m/C\(^2\))

Magnetic susceptibility (\( \chi_m \))

\[ \chi_m = \frac{M}{H} \]

\[ \chi_m = \mu_r - 1 \]
Table 20.1 Magnetic Units and Conversion Factors for the SI and cgs–emu Systems

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>SI Units</th>
<th>cgs–emu Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic induction</td>
<td>$B$</td>
<td>derived $\text{tesla} \ (\text{Wb/m}^2)^a$</td>
<td>primary $\text{kg/s-C}$</td>
<td>gauss 1 Wb/m$^2 = 10^4$ gauss</td>
</tr>
<tr>
<td>Magnetic field strength</td>
<td>$H$</td>
<td>amp-turn/m</td>
<td>primary $\text{C/m-s}$</td>
<td>oersted 1 amp-turn/m = $4\pi \times 10^{-3}$ oersted</td>
</tr>
<tr>
<td>Magnetization</td>
<td>$M$ (SI) $I$ (cgs–emu)</td>
<td>amp-turn/m</td>
<td>primary $\text{C/m-s}$</td>
<td>maxwell/cm$^2$ 1 amp-turn/m = $10^{-3}$ maxwell/cm$^2$</td>
</tr>
<tr>
<td>Permeability of a vacuum</td>
<td>$\mu_0$</td>
<td>primary $\text{henry/m}^b$</td>
<td>primary $\text{kg-m/C}^2$</td>
<td>Unitless $\frac{4\pi}{1} = 4\pi \times 10^{-7}$ henry/m = 1 emu</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>$\mu_r$ (SI) $\mu’$ (cgs–emu)</td>
<td>Unitless</td>
<td>Unitless</td>
<td>Unitless $\mu_r = \mu’$</td>
</tr>
<tr>
<td>Susceptibility</td>
<td>$\chi_m$ (SI) $\chi’_m$ (cgs–emu)</td>
<td>Unitless</td>
<td>Unitless</td>
<td>$\chi_m = 4\pi \chi’_m$</td>
</tr>
</tbody>
</table>

$a$ Units of the weber (Wb) are volt-seconds.

$b$ Units of the henry are webers per ampere.

$$\mu_0 = 4\pi \times 10^{-7} \, \text{henry.m}^{-1}$$
Magnetic Materials

Magnetic behavior is determined primarily by the electronic structure of a material, which provides magnetic dipoles.

Magnetic Dipoles: (Analogous to electric dipoles) They are the result of (a) electrons orbiting around the nucleus and (b) spin of the electron around its axis.

**Figure 20.4** Demonstration of the magnetic moment associated with (a) an orbiting electron and (b) a spinning electron.
These two motions (i.e. orbital and spin) contribute to the magnetic behavior of the material.

The interaction between these dipoles determine the type of magnetic behavior of the material.

The magnetic behavior can be controlled by composition, microstructure and processing.

The magnetic moment of an electron due to its spin is known as the **Bohr Magneton** ($M_B$ – **Fundamental Constant**)

$q=$charge of electron, 
$h=$Planck constant 
$m_e=$mass of the electron

$$M_B = \frac{qh}{4\pi m_e} = 9.274 \times 10^{-24} \text{ A.m}^2$$

Then, we can view electrons as small elementary magnets.

However, the magnetic moments due to electron do not all line up in the same direction.
Two mechanisms to cancel magnetic dipole moments:
(1) Electron pairs have opposite spins – they cancel each other.
(2) Orbital moments of the electrons also cancel out
Thus:
Atoms having completely filled electron shells (He, Ne, Ar, etc) are not capable of being permanently magnetized.

Some elements, such as transition elements (3d, 4d, 5d partially filled) have a net magnetic moment since some of their levels have unpaired electrons. Example (Sc to Cu) the electrons in the 3d level do not enter the shells in pairs. Mn has five electrons with the same spin. Transition metals have a permanent magnetic moment, which is related to the number of unpaired electrons.
<table>
<thead>
<tr>
<th>Atomic Number</th>
<th>Element</th>
<th>Partial Orbital Diagram (4s, 3d, and 4p Sublevels Only)</th>
<th>Full Electron Configuration</th>
<th>Condensed Electron Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>K</td>
<td>4s ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^1</td>
<td>[Ar] 4s^1</td>
</tr>
<tr>
<td>20</td>
<td>Ca</td>
<td>4s ▲ 3d ▼</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2</td>
<td>[Ar] 4s^2</td>
</tr>
<tr>
<td>21</td>
<td>Sc</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^1</td>
<td>[Ar] 4s^2 3d^1</td>
</tr>
<tr>
<td>22</td>
<td>Ti</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^2</td>
<td>[Ar] 4s^2 3d^2</td>
</tr>
<tr>
<td>23</td>
<td>V</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^3</td>
<td>[Ar] 4s^2 3d^3</td>
</tr>
<tr>
<td>24</td>
<td>Cr</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^1 3d^5</td>
<td>[Ar] 4s^1 3d^5</td>
</tr>
<tr>
<td>25</td>
<td>Mn</td>
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<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^5</td>
<td>[Ar] 4s^2 3d^5</td>
</tr>
<tr>
<td>26</td>
<td>Fe</td>
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<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^6</td>
<td>[Ar] 4s^2 3d^6</td>
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<td>Co</td>
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<td>[Ar] 4s^2 3d^7</td>
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<td>Ni</td>
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<td>[Ar] 4s^2 3d^8</td>
</tr>
<tr>
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<td>Cu</td>
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<td>[Ar] 4s^1 3d^10</td>
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<td>Zn</td>
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<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10</td>
<td>[Ar] 4s^2 3d^10</td>
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<tr>
<td>31</td>
<td>Ga</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^1</td>
<td>[Ar] 4s^2 3d^10 4p^1</td>
</tr>
<tr>
<td>32</td>
<td>Ge</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^2</td>
<td>[Ar] 4s^2 3d^10 4p^2</td>
</tr>
<tr>
<td>33</td>
<td>As</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^3</td>
<td>[Ar] 4s^2 3d^10 4p^3</td>
</tr>
<tr>
<td>34</td>
<td>Se</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^4</td>
<td>[Ar] 4s^2 3d^10 4p^4</td>
</tr>
<tr>
<td>35</td>
<td>Br</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^5</td>
<td>[Ar] 4s^2 3d^10 4p^5</td>
</tr>
<tr>
<td>36</td>
<td>Kr</td>
<td>4s ▲ 3d ▼ ▲</td>
<td>[1s^2 2s^2 2p^6 3s^2 3p^6] 4s^2 3d^10 4p^6</td>
<td>[Ar] 4s^2 3d^10 4p^6</td>
</tr>
</tbody>
</table>

*Colored type indicates sublevel(s) whose occupancy changes when last electron is added.*
Types of magnetism:

- **Ferromagnetism.** Property of iron, nickel, neodymium. Strongest type of magnetism. Uncancelled electron spins as a consequence of the electron structure.
- **Paramagnetism.** Exhibited by materials containing transition, rare earth or actinide elements.
- **Diamagnetism.** Exhibited by all common materials but masked if other two types of magnetism are present.
- **Ferrimagnetism.** Source of magnetic moment different as ferromagnetic. Incomplete cancellation of spin moments due to atomic position and surrounding. Ceramics (insulators).
- **Antiferromagnetism.** Alignment of the spin moments of neighboring atoms or ions in exactly opposite direction.
Figure 20.6  Schematic representation of the flux density $B$ versus the magnetic field strength $H$ for diamagnetic, paramagnetic, and ferromagnetic materials.
Diamagnetism

Completely filled shells or subshells.
Total cancellation of orbital and spin moments
Cannot be permanently magnetized. Very weak
It is induced by change in orbital motion due to applied field
The dipoles induced by the field are aligned opposite to the field direction.
Only exists while a field is on. It is found in all materials
Very hard to observe. It is of no practical purpose

\[ \mu_r < 1 \text{ (} \sim 0.99 \text{)} \]

\[ \mu_r = -1 \]

\[ H = 0 \]
Paramagnetism

Incomplete cancellation of electron spin/orbital magnetic moments

Permanent Dipoles. Randomly oriented when no field is present

Paramagnetism. Permanent dipoles align with an external field

No interaction between adjacent dipoles. Exists only in a magnetic field

Randomly oriented permanent dipoles align with field. (Only present when field is applied)

\[ \mu_r = 1.00 \text{ to } 1.01 \]

\( H = 0 \)
<table>
<thead>
<tr>
<th>Material</th>
<th>Susceptibility $\chi_m$ (x 10^{-5})</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diamagnetic</strong></td>
<td></td>
</tr>
<tr>
<td>Bismuth</td>
<td>-16.6</td>
</tr>
<tr>
<td>Mercury</td>
<td>-2.85</td>
</tr>
<tr>
<td>Silver</td>
<td>-2.38</td>
</tr>
<tr>
<td>Carbon (diamond)</td>
<td>-2.1</td>
</tr>
<tr>
<td>Gold</td>
<td>-3.44</td>
</tr>
<tr>
<td>Sodium chloride</td>
<td>-1.4</td>
</tr>
<tr>
<td>Copper</td>
<td>-1.8</td>
</tr>
<tr>
<td><strong>Paramagnetic</strong></td>
<td></td>
</tr>
<tr>
<td>Iron aluminum alum</td>
<td>66</td>
</tr>
<tr>
<td>Uranium</td>
<td>40</td>
</tr>
<tr>
<td>Platinum</td>
<td>26</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.07</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.85</td>
</tr>
<tr>
<td>Chromium</td>
<td>3.13</td>
</tr>
</tbody>
</table>

\[ \mu = (1.00 \rightarrow 1.01) \mu_0 \] for paramagnetic solid

\[ \mu_0 \] for vacuum

\[ \mu \approx 0.99999 \mu_0 \] for diamagnetic solid
**Ferromagnetism**

- Electron Spins don’t cancel out
- Coupling interactions cause adjacent atoms to align with one another
- Ferromagnetism. Permanent magnetic moment in the absence of an external field.
  - large magnetization
  - $\mu_r = \text{up to } 10^6$

Permanent dipoles are aligned even in the absence of a magnetic field.

$H = 0$
As a whole the material’s magnetic domains are oriented randomly and effectively cancel each other out.

If $H$ is applied, domains align giving a strong net $H$ field in same direction as $H$.

Net $H$ field partially exists even when $H_{ext}$ is removed.
If we place ferromagnetic material (e.g. iron) inside a solenoid with field $B_0$, increase the total $B$ field inside coil to

$$B = B_0 + B_M$$

$$B_0 = \mu_0 H$$

$$B_M = \mu_0 M$$

$B_M$ is magnitude of $B$ field contributed by iron core

$B_M$ result of alignment of the domains

$B_M$ increases total $B$ by large amount - iron core inside solenoid increases $B$ by typically about 5000 times

For the electromagnetic core we use “soft” iron where the magnetism is not permanent (goes away when the external field is turned off).

$$M \gg H \quad B \cong \mu_0 M$$

The maximum possible magnetization, or saturation magnetization $M_s$ represents the magnetization that results when all the magnetic dipoles in a solid piece are mutually aligned with the external field. There is also a corresponding saturation flux density $B_s$
**Figure 20.11**  Schematic depiction of domains in a ferromagnetic or ferrimagnetic material; arrows represent atomic magnetic dipoles. Within each domain, all dipoles are aligned, whereas the direction of alignment varies from one domain to another.

**Fig. 15.16.** Domain pattern observed in a (001) iron single crystal film of thickness 2500 Å, and its diffraction pattern (inset).
**B small:** Domains with favorable orientation to applied field grow at the expense of others.

**B large:** Rotation of the domain orientation towards the applied field.

**M_s:** Saturation Magnetization

**Figure 20.13** The $B$-versus-$H$ behavior for a ferromagnetic or ferrimagnetic material that was initially unmagnetized. Domain configurations during several stages of magnetization are represented. Saturation flux density $B_s$, magnetization $M_s$, and initial permeability $\mu_i$ are also indicated. (Adapted from O. H. Wyatt and D. Dew-Hughes, *Metals, Ceramics and Polymers*, Cambridge University Press, 1974.)
Example

The maximum magnetization, called \textit{saturation magnetization} $M_{\text{SAT}}$, in iron is about $1.75 \times 10^6 \text{ Am}^{-1}$. This corresponds to all possible net spins aligning parallel to each other. Calculate the effective number of \textit{Bohr magnetons} per atom that would give $M_{\text{SAT}}$, given that the density and relative atomic mass of iron are $7.86 \text{ g.cm}^{-3}$ and $55.85 \text{ g.mol}^{-1}$ respectively.

\textbf{Solution:}

Number of iron atoms per unit volume

$$n_{\text{Fe}} = \frac{\rho N_{\text{Avogadro}}}{A_{\text{Fe}}} = \frac{(7.86 \times 10^3 \text{ kg.m}^{-3})(6.022 \times 10^{23} \text{ mol}^{-1})}{55.85 \times 10^3 \text{ kg.mol}^{-1}}$$

$$n_{\text{Fe}} = 8.48 \times 10^{28} \text{ atoms.m}^{-3}$$

The magnetic saturation is given by the expression:

Where $\zeta$ is the number of net spins that contribute to magnetization per iron atom

$$M_{\text{SAT}} = \zeta \times n_{\text{Fe}} \times M_B$$

$$\zeta = \frac{M_{\text{SAT}}}{n_{\text{Fe}} \times M_B} = 2.2$$
<table>
<thead>
<tr>
<th></th>
<th>Fe</th>
<th>Co</th>
<th>Ni</th>
<th>Gd</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Crystal structure</strong></td>
<td>BCC</td>
<td>HCP</td>
<td>FCC</td>
<td>HCP</td>
</tr>
<tr>
<td><strong>Bohr magnetons per atom</strong></td>
<td>2.22</td>
<td>1.72</td>
<td>0.60</td>
<td>7.1</td>
</tr>
<tr>
<td><strong>M_{sat}(0) (MA m^{-1})</strong></td>
<td>1.75</td>
<td>1.45</td>
<td>0.50</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>B_{sat} = \mu_o M_{sat} (T)</strong></td>
<td>2.2</td>
<td>1.82</td>
<td>0.64</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>T_C</strong></td>
<td>770 °C</td>
<td>1127 °C</td>
<td>358 °C</td>
<td>16 °C</td>
</tr>
<tr>
<td></td>
<td>1043 K</td>
<td>1400 K</td>
<td>631 K</td>
<td>289 K</td>
</tr>
</tbody>
</table>

Example

Calculate the maximum, or saturation magnetization and the saturation flux density that we expect in iron. The lattice parameter of BCC iron is 2.866\text{Angstroms}. Data Fe (Z=26)

Solution

Based on the unpaired electronic spins (see table), we expect each iron atom to have 2.22 electrons that act as magnetic dipoles.

\[ \text{Number of iron atoms per m}^3 = \frac{2 \text{ atoms per cell}}{(2.866 \times 10^{-10} \text{ m})^3} = 8.48 \times 10^{28} \]

The maximum volume magnetization (\(M_{\text{SAT}}\)) is the total magnetic moment per unit volume

\[ M_{\text{SAT}} = \left(8.48 \times 10^{28} \text{ atoms} / \text{m}^3\right) \left(9.27 \times 10^{-24} \text{ A.m}^2\right) \left(2.22\text{ Bohr Magnetons/atom}\right) \]

\[ M_{\text{SAT}} = 1.75 \times 10^6 \text{ A.m}^{-1} \]

In ferromagnetic materials \(\mu_0M \gg \mu_0H\) and therefore \(B \sim \mu_0M\)

\[ B_{\text{SAT}} = \left(4\pi \times 10^{-7} \frac{\text{Wb}}{\text{m.A}}\right) \left(1.75 \times 10^6 \frac{\text{A}}{\text{m}}\right) = 2.19 \frac{\text{Wb}}{\text{m}^2} = 2.19 \text{ Tesla} \]
Example

Calculate (a) the saturation magnetization and (b) the saturation flux density for nickel, which has a density of 8.90g.cm\(^{-3}\). Data \(A_{\text{Ni}} = 58.69\text{g.mol}^{-1}\)

Solution

(a) Based on the table, we expect each nickel atom to have 0.6 electrons that act as magnetic dipoles. The number of nickel atoms per cubic meter is related to the density of the nickel

\[
N = \frac{\rho N_A}{A_{\text{Ni}}} = 9.13 \times 10^{28} \text{ atoms per m}^3
\]

The saturation magnetization is

\[
M_{\text{SAT}} = 0.60 \times \text{Bohr Magneton} \times N = 5.1 \times 10^5 \text{ A.m}^{-1}
\]

(b) Flux density :

\[
B_{\text{SAT}} = \mu_0 M_{\text{SAT}} = 0.64 \text{ Tesla}
\]
Ferrimagnetism

- Ceramics. Permanent magnetism
  (Source of net magnetic moment is different)
- Cubic ferrites $M\text{Fe}_2\text{O}_4$ ($M = \text{Ni, Mn, Co, Cu}$)
- Can adjust composition to get different properties
- Ferrites are usually good electronic insulators

![Diagram of ferrimagnetism]

### Table 20.3 The Distribution of Spin Magnetic Moments for $\text{Fe}^{2+}$ and $\text{Fe}^{3+}$ Ions in a Unit Cell of $\text{Fe}_3\text{O}_4$

<table>
<thead>
<tr>
<th>Cation</th>
<th>Octahedral Lattice Site</th>
<th>Tetrahedral Lattice Site</th>
<th>Net Magnetic Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Fe}^{3+}$</td>
<td>$\uparrow\uparrow\uparrow\uparrow$</td>
<td>$\downarrow\downarrow\downarrow\downarrow$</td>
<td>Complete cancellation</td>
</tr>
<tr>
<td>$\text{Fe}^{2+}$</td>
<td>$\uparrow\uparrow\uparrow\uparrow$</td>
<td>$-$</td>
<td></td>
</tr>
</tbody>
</table>

$a$ Each arrow represents the magnetic moment orientation for one of the cations.

**Figure 20.9** Schematic diagram showing the spin magnetic moment configuration for $\text{Fe}^{2+}$ and $\text{Fe}^{3+}$ ions in $\text{Fe}_3\text{O}_4$. (From Richard A. Flinn and Paul K. Trojan, *Engineering Materials and Their Applications*, 4th edition. Copyright © 1990 by John Wiley & Sons, Inc. Adapted by permission of John Wiley & Sons, Inc.)
Cubic ferrites having other compositions may be produced by adding metallic ions that substitute for some of the iron in the crystal structure.

Again, from the ferrite chemical formula, \( M^{2+}O^{2-}(Fe^{3+})_2(O^2^-)_3 \), in addition to \( Fe^{2+} \), \( M^{2+} \) may represent divalent ions such as \( Ni^{2+} \), \( Mn^{2+} \), \( Co^{2+} \), and \( Cu^{2+} \), each of which possesses a net spin magnetic moment different from 4; several are listed in Table 18.4.

Thus, by adjustment of composition, ferrite compounds having a range of magnetic properties may be produced. For example, nickel ferrite has the formula \( NiFe_2O_4 \).

Other compounds may also be produced containing mixtures of two divalent metal ions such as \( (Mn,Mg)Fe_2O_4 \), in which the \( Mn^{2+}:Mg^{2+} \) ratio may be varied; these are called mixed ferrites.

<table>
<thead>
<tr>
<th>Cation</th>
<th>Net Spin Magnetic Moment (Bohr magnetons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe(^{3+})</td>
<td>5</td>
</tr>
<tr>
<td>Fe(^{2+})</td>
<td>4</td>
</tr>
<tr>
<td>Mn(^{2+})</td>
<td>5</td>
</tr>
<tr>
<td>Co(^{2+})</td>
<td>3</td>
</tr>
<tr>
<td>Ni(^{2+})</td>
<td>2</td>
</tr>
<tr>
<td>Cu(^{2+})</td>
<td>1</td>
</tr>
</tbody>
</table>
Example

Calculate the saturation magnetization for Fe₃O₄ given that each cubic unit cell contains 8 Fe²⁺ and 16 Fe³⁺ ions and that the lattice parameter is 0.839nm.

Solution

The net magnetization results from the Fe²⁺ ions only. Since there are 8 Fe²⁺ ions per unit cell and 4 Bohr magneton per Fe²⁺ ion, then the number of electron contributing to the dipole magnetization per unit cell is 32 (nₜ). 

\[
M_{SAT} = \frac{n_B \times \text{Bohr magneton}}{a^3}
\]

\[
M_{SAT} = \frac{(32 \_ \text{Bohr \_ Magneton \_ per \_ unit \_ cell})(9.27 \times 10^{-24} \text{ A.m}^2 / \text{Bohr \_ magneton})}{(0.839 \times 10^{-9} \text{ m})^3 \_ per \_ unit \_ cell}
\]

\[
M_{SAT} = 5.0 \times 10^5 \text{ A.m}^{-1}
\]
Example

Design a cubic mixed-ferrite magnetic material that has a saturation magnetization of \(5.25 \times 10^5\) A.m\(^{-1}\)

Solution

From the previous example the, if all were Fe\(^{2+}\), the saturation magnetization should be \(5.0 \times 10^5\) A.m\(^{-1}\). Then, some of the iron Fe\(^{2+}\) ions must be replaced by other ions with more than 4 Bohr Magnetons per ions, such as Mn.

\[
n_B = \frac{M_{\text{SAT}} \times a^3}{\text{Bohr magneton}} = 33.45 \text{ Bohr Magneton per unit cell}
\]

If \(x\) represents the fraction of Mn\(^{2+}\) that have substitute for Fe\(^{2+}\), then the remaining unsubstituted Fe\(^{2+}\) fraction is \((1-x)\)

\[
8[5x + 4(1 - x)] = 33.45 \quad \text{Or 18.1}\%\text{Mn}
\]

\[x = 0.181\]
Antiferromagnetism
Alignment of neighboring atoms
Spin moments are opposite
no net magnetic moment

Antiparallel alignment
Ceramic oxides