Spherical Pressure Vessels

Pressure vessels are closed structures containing liquids or gases under pressure. Examples include tanks, pipes, pressurized cabins, etc.

Shell structures: When pressure vessels have walls that are thin in comparison to their radii and length. In the case of thin walled pressure vessels of spherical shape the ratio of radius $r$ to wall thickness $t$ is greater than 10. A sphere is the theoretical ideal shape for a vessel that resists internal pressure.

To determine the stresses in an spherical vessel let us cut through the sphere on a vertical diameter plane and isolate half of the shell and its fluid contents as a single free body. Acting on this free body are the tensile stress $\sigma$ in the wall of the vessel and the fluid pressure $p$. 

![Diagram of a spherical vessel with arrows indicating stress and pressure](image)
The pressure that acts horizontally against the plane circular area is uniform and gives a resultant pressure force of:

Where \( p \) is the gage or internal pressure (above the pressure acting in the outside of the vessel).

The stress is uniform around the circumference and it is uniformly distributed across the thickness \( t \) (because the wall is thin). The resultant horizontal force is:

Equilibrium of forces in the horizontal direction:

\[
P = p \pi r^2
\]

\[
\sigma (2 \pi r_m) t = p \pi r^2
\]

\[
\sigma = \frac{pr}{2t}
\]

\( r_m \sim r \) for thin walls. Therefore the formula to calculate the stress in a thin walled spherical vessels is:
As is evident from the symmetry of a spherical shell that we will obtain the same equation regardless of the direction of the cut through the center.

*The wall of a pressurized spherical vessel is subjected to uniform tensile stresses $\sigma$ in all directions.*

Stresses that act tangentially to the curved surface of a shell are known as *membrane stresses*.

**Limitations of the thin-shell theory:**

1. The wall thickness must be small ($r/t > 10$)
2. The internal pressure must exceed the external pressure.
3. The analysis is based only on the effects of internal pressure.
4. The formulas derived are valid throughout the wall of the vessel except near points of stress concentration.
Stresses at the Outer Surfaces.

The element below has the \( x \) and \( y \) axes tangential to the surface of the sphere and the \( z \) axis is perpendicular to the surface. Thus, the normal stresses \( \sigma_x \) and \( \sigma_y \) are equal to the membrane stress \( \sigma \) and the normal stress \( \sigma_z \) is zero.

The principal stresses are

\[
\sigma_1 = \sigma_2 = \sigma = \frac{pr}{2t}
\]

and \( \sigma_3 = 0 \). Any rotation element about the \( z \) axis will have a shear stress equals to \textit{zero}.

To obtain the maximum shear stresses, we must consider out of plane rotations, that is, rotations about the \( x \) and \( y \) axis.

Elements oriented at \( 45^\circ \) of the \( x \) or \( y \) axis have maximum shear stresses equal to \( \sigma/2 \) or

\[
\tau_{\text{Max}} = \frac{\sigma}{2} = \frac{pr}{4t}
\]
Stresses at the Inner Surface

At the inner wall the stresses in the \( x \) and \( y \) direction are equal to the membrane stress \( \sigma_x = \sigma_y = \sigma \), but the stress in the \( z \) direction is not \textit{zero}, and it is equal to the pressure \( p \) in compression.

This compressive stress decreases from \( p \) at the inner surface to \textit{zero} at the outer surface.

The element is in \textit{triaxial} stress

\[
\sigma_1 = \sigma_2 = \sigma = \frac{pr}{2t} \\
\sigma_3 = -p
\]

The in-plane shear stress are \textit{zero}, but the maximum out-of-plane shear stress (obtained at 45\(^\circ\) rotation about either the \( x \) or \( y \) axis) is

\[
\tau_{Max} = \frac{(\sigma + p)}{2} = \frac{pr}{4t} + \frac{p}{2} \\
\tau_{Max} = \frac{p}{2} \left( 1 + \frac{r}{2t} \right)
\]
When the vessels is thin walled and the ratio $r/t$ is large, we can disregard the number 1 and

$$\tau_{\text{Max}} = \frac{pr}{4t}$$

Consequently, we can consider the stress state at the inner surface to be the same as the outer surface.

**Summary for Spherical pressure vessel with r/t large:**

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$
As the two stresses are equal, Mohr’s circle for in-plane transformations reduces to a point

\[ \sigma = \sigma_1 = \sigma_2 = \text{constant} \]

\[ \tau_{\text{max (in-plane)}} = 0 \]

Maximum out-of-plane shearing stress

\[ \tau_{\text{max}} = \frac{1}{2} \sigma_1 = \frac{pr}{4t} \]
Example

A compressed air tank having an inner diameter of 18 inches and a wall thickness of \( \frac{1}{4} \) inch is formed by welding two steel hemispheres (see figure).

(a) If the allowable tensile stress in the steel is 14000 psi, what is the maximum permissible air pressure \( p_a \) in the tank?

(b) If the allowable shear stress in the steel is 6000 psi, what is the maximum permissible pressure \( p_b \)?

(c) If the normal strain in the outer surface of the tank is not to exceed 0.0003, what is the maximum permissible pressure \( p_c \)? (Assume Hooke’s law is obeyed \( E = 29 \times 10^6 \) psi and Poisson’s ratio is \( \nu = 0.28 \))

(d) Tests on the welded seam show that failure occurs when the tensile load on the welds exceeds 8.1 kips per inch of weld. If the required factor of safety against failure of the weld is 2.5, what is the maximum permissible pressure \( p_d \)?

(e) Considering the four preceding factors, what is the allowable pressure \( p_{allow} \) in the tank?
Solution

(a) Allowable pressure based on the tensile stress in the steel. We will use the equation

\[ \sigma_{\text{allowed}} = \frac{p_ar}{2t} \]  
then

\[ p_a = \frac{2t \sigma_{\text{allowed}}}{r} = \frac{2(0.25\text{inch})(14000\text{psi})}{9.0\text{inch}} = 777.8\text{psi} \]

(b) Allowable pressure based upon the shear stress of the steel. We will use

\[ \tau_{\text{allowed}} = \frac{\sigma}{2} = \frac{p_br}{4t} \]  
then

\[ p_b = \frac{4t \tau_{\text{allowed}}}{r} = \frac{4(0.25\text{inch})(6000\text{psi})}{9.0\text{inch}} = 666.7\text{psi} \]

(c) Allowable pressure based upon the normal strain in the steel. For biaxial stress

\[ \varepsilon_x = \frac{(\sigma_x - \nu \sigma_y)}{E} \]  
substituting \( \sigma_x = \sigma_y = \sigma = \frac{pr}{2t} \)
then
\[ \varepsilon_x = \frac{(1-\nu)\sigma}{E} = \frac{(1-\nu)pr}{2tE} \]

this equation can be solved for pressure \( p_c \)

\[ p_c = \frac{2tE \varepsilon_{\text{allowed}}}{r(1-\nu)} = \frac{2(0.25\text{inch})(29 \times 10^6\text{ psi})(0.0003)}{(9.0\text{inch})(1-0.28)} = 671.3\text{psi} \]
(d) Allowable pressure based upon the tension in the welded seam.

The allowable tensile load on the welded seam is equal to the failure load divided by the factor of safety:

\[ T_{\text{allowed}} = \frac{T_{\text{failure}}}{n} = \frac{8.1\text{kips/inch}}{2.5} = 3.24\text{kips/inch} = 3240\text{lb/inch} \]

The corresponding allowable tensile stress is equal to the allowable load on 1 inch length of weld divided by the cross-sectional area of a 1 inch length of weld:

\[ \sigma_{\text{allowed}} = \frac{T_{\text{allowed}}}{(1.0\text{inch})(t)} = \frac{3240\text{ lb/inch}(1.0\text{inch})}{(1.0\text{inch})(0.25\text{inch})} = 12960\text{ psi} \]

\[ p_d = \frac{2t\sigma_{\text{allowed}}}{r} = \frac{2(0.25\text{inch})(12960\text{psi})}{9.0\text{inch}} = 720.0\text{ psi} \]

(e) Allowable pressure

Comparing the preceding results for \( p_a, p_b, p_c \) and \( p_d \), we see that the shear stress in the wall governs and the allowable pressure in the tank is \( p_{\text{allow}} = 666\text{psi} \).
Examples: Compressed air tanks, rocket motors, fire extinguishers, spray cans, propane tanks, grain silos, pressurized pipes, etc.

We will consider the normal stresses in a thin walled circular tank $AB$ subjected to internal pressure $p$.

$\sigma_1$ and $\sigma_2$ are the membrane stresses in the wall. No shear stresses act on these elements because of the symmetry of the vessel and its loading, therefore $\sigma_1$ and $\sigma_2$ are the principal stresses.

Because of their directions, the stress $\sigma_1$ is called circumferential stress or the hoop stress, and the stress $\sigma_2$ is called the longitudinal stress or the axial stress.
This longitudinal stress is equal to the membrane stress in a spherical vessel. Then:

\[ \sigma_1 (2b)t = p (2b)r \]
\[ \sigma_1 = \frac{pr}{t} \]

Equilibrium of forces to find the longitudinal stress:

\[ \sigma_2 (2\pi r)t = p (\pi r^2) \]
\[ \sigma_2 = \frac{pr}{2t} \]

We note that the longitudinal welded seam in a pressure tank must be twice as strong as the circumferential seam.
Stresses at the Outer Surface

The principal stresses $\sigma_1$ and $\sigma_2$ at the outer surface of a cylindrical vessel are shown below. Since $\sigma_3$ is zero, the element is in biaxial stress. The maximum in plane shear stress occurs on planes that are rotated $45^\circ$ about the $z$-axis.

\[
(\tau_{Max})_z = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{pr}{4t}
\]

The maximum out of plane shear stresses are obtained by $45^\circ$ rotations about the $x$ and $y$ axes respectively.

\[
\tau_{Max,x} = \frac{\sigma_1}{2} = \frac{pr}{2t}
\]

\[
\tau_{Max,y} = \frac{\sigma_2}{2} = \frac{pr}{4t}
\]

Then, the absolute maximum shear stress is $\tau_{max} = \frac{pr}{2t}$, which occurs on a plane that has been rotated $45^\circ$ about the $x$-axis.
Stresses at the Inner Surface

The principal stresses are

\[ \sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{pr}{2t} \quad \text{and} \quad \sigma_3 = -p \]

The three maximum shear stresses, obtained by \(45^\circ\) rotations about the \(x\), \(y\) and \(z\) axes are

\[ (\tau_{\text{MAX}})_x = \frac{(\sigma_1 - \sigma_3)}{2} = \frac{pr}{2t} + \frac{p}{2} \]
\[ (\tau_{\text{Max}})_y = \frac{(\sigma_2 - \sigma_3)}{2} = \frac{pr}{4t} + \frac{p}{2} \]
\[ (\tau_{\text{Max}})_z = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{pr}{4t} \]

The first of these three stresses is the largest. However, when \(r/t\) is very large (thin walled), the term \(p/2\) can be disregarded, and the equations are the same as the stresses at the outer surface.
Summary for Cylindrical vessels with \( r/t \) large

Cylindrical vessel with principal stresses

\[ \sigma_1 = \text{hoop stress} \]
\[ \sigma_2 = \text{longitudinal stress} \]

Hoop stress:

\[ \sum F_z = 0 = \sigma_1 (2t \Delta x) - p(2r \Delta x) \]
\[ \sigma_1 = \frac{pr}{t} \]

Longitudinal stress:

\[ \sum F_x = 0 = \sigma_2 (2\pi rt) - p(\pi r^2) \]
\[ \sigma_2 = \frac{pr}{2t} \]
\[ \sigma_1 = 2\sigma_2 \]
Points \( A \) and \( B \) correspond to hoop stress, \( \sigma_1 \), and longitudinal stress, \( \sigma_2 \)

Maximum in-plane shearing stress:

\[
\tau_{\text{max (in-plane)}} = \frac{1}{2} \sigma_2 = \frac{pr}{4t}
\]

Maximum out-of-plane shearing stress corresponds to a 45° rotation of the plane stress element around a longitudinal axis

\[
\tau_{\text{max}} = \sigma_2 = \frac{pr}{2t}
\]
A cylindrical pressure vessel is constructed from a long, narrow steel plate by wrapping the plate around a mandrel and then welding along the edges of the plate to make an helical joint (see figure below). The helical weld makes an angle $\alpha = 55^\circ$ with the longitudinal axis. The vessel has an inner radius $r = 1.8\, \text{m}$ and a wall thickness $t = 20\, \text{mm}$. The material is steel with a modulus $E = 200\, \text{GPa}$ and a Poisson’s ratio $\nu = 0.30$. The internal pressure $p$ is $800\, \text{kPa}$.

Calculate the following quantities for the cylindrical part of the vessel:

The circumferential and longitudinal stresses $\sigma_1$ and $\sigma_2$ respectively;

The maximum in-plane and out-of-plane shear stresses

The circumferential and longitudinal strains $\varepsilon_1$ and $\varepsilon_2$ respectively, and

The normal stress $\sigma_w$ and shear stress $\tau_w$ acting perpendicular and parallel, respectively, to the welded seam.
Solution

Circumferential and longitudinal stresses:

(b) Maximum Shear Stress

The largest in-plane shear stress is obtained from the equation

\[ \tau_{\text{Max}} \text{ in-plane} = \frac{\sigma_1 - \sigma_2}{2} = \frac{pr}{4t} = 18 \text{ MPa} \]

The largest out-of-plane shear stress is obtained from the equation:

\[ \tau_{\text{Max}} \text{ out-of-plane} = \frac{\sigma_1}{2} = \frac{pr}{t} = 36 \text{ MPa} \]

This last stress is the absolute maximum shear stress in the wall of the vessel.

(c) Circumferential and longitudinal strains.

Assume the Hook’s law applies to the wall of the vessel. Using the equations

\[ \epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} \]
\[ \epsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \]

We note that \( \epsilon_x = \epsilon_2 \) and \( \epsilon_y = \epsilon_1 \) and also \( \sigma_x = \sigma_2 \) and \( \sigma_y = \sigma_1 \). Therefore the above equations can be written in the following forms:
\[ \varepsilon_2 = \frac{(1-2\nu)pr}{2tE} = \frac{(1-2\nu)\sigma_2}{E} = \frac{(1-2(0.30))(36 \text{ MPa})}{200 \text{ GPa}} = 72 \times 10^{-6} \]

\[ \varepsilon_1 = \frac{(2-\nu)pr}{2tE} = \frac{(2-\nu)\sigma_1}{2E} = \frac{(2-0.30)(72 \text{ MPa})}{(2)(200 \text{ GPa})} = 306 \times 10^{-6} \]

(d) Normal and shear stresses acting on the weld seam

The angle \( \theta \) for the stress element at point \( B \) in the wall of the cylinder with sides parallel and perpendicular to the weld is

\[ \theta = 90^\circ - \alpha = 35^\circ \]

We will use the stress transformation equations:

\[ \sigma_{x1} = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta \]

\[ \tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \]

\[ \sigma_{x1} + \sigma_{y1} = \sigma_x + \sigma_y \]
\[ \sigma_{x_1} = 36(\cos(35))^2 + 72(\sin(35))^2 = 47.8 \text{MPa} \]

\[ \tau_{x_1y_1} = -36 \cdot \sin(35) \cdot \cos(35) + 72 \cdot \sin(35) \cdot \cos(35) = 16.9 \text{MPa} \]

\[ 47.8 + \sigma_{y_1} = 36 + 72 \Rightarrow \sigma_{y_1} = 60.2 \text{MPa} \]

**Mohr’s circle**

Coordinates of point A (for \( \theta = 0 \)) \( \sigma_2 = 36 \text{MPa} \)
and shear stress = 0
Coordinates of point B (for \( \theta = 90 \)) \( \sigma_1 = 72 \text{MPa} \)
and shear stress = 0
Center (point C) = \( (\sigma_1 + \sigma_2) / 2 = 54 \text{MPa} \)
Radius = \( (\sigma_1 - \sigma_2) / 2 = 18 \text{MPa} \)

A counterclockwise angle \( 2\theta = 70^o \) (measured on the circle from point A)
locates point D, which corresponds to the stresses on the \( x_1 \) face (\( \theta = 35^o \)) of
the element.

The coordinates of point D
\[ \sigma_{x_1} = 54 - R \cos 70^o = 54 \text{MPa} - (18 \text{MPa})(\cos 70^o) = 47.8 \text{MPa} \]
\[ \tau_{x_1y_1} = R \sin 70^o = (18 \text{MPa})(\sin 70^o) = 16.9 \text{MPa} \]
When seen in a side view, a **helix** follows the shape of a sine curve (see Figure below). The pitch of the helix is \( p = \pi d \tan \theta \), where \( d \) is the diameter of the circular cylinder and \( \theta \) is the angle between a normal to the helix and a longitudinal line. The width of the flat plate that wraps into the cylinder shape is \( w = \pi d \sin \theta \).

For practical reasons, the angle \( \theta \) is usually in the range from **20° to 35°**.
Quiz

A close-end pressure vessel has an inside diameter of 1600 mm and a wall thickness of 40mm. It is pressurized to an internal pressure of 6.5MPa and it has a centric compressive force of 750kN applied as shown. The tank is welded together along the helix making an angle of 25° to the horizontal. Determine the normal and shear stresses along the helix.

The material is steel with a modulus $E = 200\text{GPa}$ and a Poisson’s ratio $\nu = 0.30$.

Calculate the following quantities for the cylindrical part of the vessel:

(a) The circumferential and longitudinal stresses $\sigma_1$ and $\sigma_2$ respectively;

(b) The maximum in-plane and out-of-plane shear stresses;

(c) The circumferential and longitudinal strains $\varepsilon_1$ and $\varepsilon_2$ respectively, and

(d) The normal stress $\sigma_w$ and shear stress $\tau_w$ acting perpendicular and parallel, respectively, to the welded seam.
**General State of Stresses**

From equilibrium principles:
\[ \tau_{xy} = \tau_{yx}, \quad \tau_{xz} = \tau_{zx}, \quad \tau_{yz} = \tau_{zy} \]

The most general state of stress at a point may be represented by 6 components

**Normal Stresses**

\[ \sigma_x, \quad \sigma_y, \quad \sigma_z \]

**Shear Stresses**

\[ \tau_{xy}, \quad \tau_{yz}, \quad \tau_{xz} \]

These are the principal axes and principal planes and the normal stresses are the principal stresses.
The three circles represent the normal and shearing stresses for rotation around each principal axis.

Points $A$, $B$, and $C$ represent the principal stresses on the principal planes (shearing stress is zero).

Radius of the largest circle yields the maximum shearing stress.

\[
\tau_{\text{max}} = \frac{1}{2} |\sigma_{\text{max}} - \sigma_{\text{min}}|
\]
In the case of plane stress, the axis perpendicular to the plane of stress is a principal axis (shearing stress equal zero).

If the points $A$ and $B$ (representing the principal planes) are on opposite sides of the origin, then

the corresponding principal stresses are the maximum and minimum normal stresses for the element

the maximum shearing stress for the element is equal to the maximum “in-plane” shearing stress

planes of maximum shearing stress are at $45^\circ$ to the principal planes.
If A and B are on the same side of the origin (i.e., have the same sign), then

the circle defining $\sigma_{\text{max}}, \sigma_{\text{min}},$ and $\tau_{\text{max}}$ for the element is not the circle corresponding to transformations within the plane of stress

maximum shearing stress for the element is equal to half of the maximum stress

planes of maximum shearing stress are at 45 degrees to the plane of stress
**Failure Theories**

*Why do mechanical components fail?* Mechanical components fail because the applied stresses exceeds the material’s strength (Too simple).

*What kind of stresses cause failure?* Under any load combination, there is always a combination of normal and shearing stresses in the material.

**Mohr’s circle for centric axial loading:**

\[
\sigma_x = \frac{P}{A}, \quad \sigma_y = \tau_{xy} = 0
\]

\[
\sigma_x = \sigma_y = \tau_{xy} = \frac{P}{2A}
\]

**Mohr’s circle for torsional loading:**

\[
\sigma_x = \sigma_y = 0 \quad \tau_{xy} = \frac{T_c}{J}
\]

\[
\sigma_x = \sigma_y = \frac{T_c}{J} \quad \tau_{xy} = 0
\]
What is the definition of Failure?
Obviously fracture but in some components yielding can also be considered as failure, if yielding distorts the material in such a way that it no longer functions properly.

Which stress causes the material to fail?
Usually ductile materials are limited by their shear strengths. While brittle materials (ductility < 5%) are limited by their tensile strengths.

Stress at which point?
Yield Criteria for Ductile Materials under Plane Stress Conditions

**Maximum Shear Stress Theory (MSST)**

Failure occurs when the maximum shear stress in the part (subjected to plane stress) exceeds the shear stress in a tensile test specimen (of the same material) at yield.

**Maximum shear stress criteria:** The structural component is safe as long as the maximum shear stress is less than the maximum shear stress in a tensile test specimen at yield, i.e.,

\[
\tau_{\text{yield}} = \frac{S_y}{2}
\]

For \(\sigma_a\) and \(\sigma_b\) with the same sign,

\[
\tau_{\text{max}} < \tau_y = \frac{\sigma_y}{2}
\]

or

\[
\frac{\sigma_a}{2} < \frac{\sigma_y}{2}
\]

For \(\sigma_a\) and \(\sigma_b\) with opposite signs,

\[
\tau_{\text{max}} = \frac{|\sigma_a - \sigma_b|}{2} < \frac{\sigma_y}{2}
\]

The maximum absolute shear stress is always the radius of the largest of the Mohr’s circle.
**Distortion Energy Theory (DET)**

Based on the consideration of angular distortion of stressed elements. The theory states that failure occurs when the distortion strain energy in the material exceeds the distortion strain energy in a tensile test specimen (of the same material) at yield.

**Maximum distortion energy criteria:** Structural component is safe as long as the distortion energy per unit volume is less than that occurring in a tensile test specimen at yield.

**Von Mises effective stress:** Defined as the uniaxial tensile stress that creates the same distortion energy as any actual combination of applied stresses.

\[
\begin{align*}
\frac{1}{6G} \left( \sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 \right) &< \frac{1}{6G} \left( \sigma_Y^2 - \sigma_Y \times 0 + 0^2 \right) \\
\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 &< \sigma_Y^2
\end{align*}
\]

\[
\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1 \sigma_2 - \sigma_2 \sigma_3 - \sigma_3 \sigma_1}
\]

\[
\sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_3 \sigma_1}
\]

\[
\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}
\]
Example

Given the material \( S_Y, \sigma_x, \sigma_y \) and \( \tau_{xy} \) find the safety factors for all the applicable criteria.

a. Pure aluminum  \( S_Y = 30\text{MPa} \quad \sigma_x = 10\text{MPa} \quad \sigma_y = -10\text{MPa} \quad \tau_{xy} = 0\text{MPa} \)

\[ \sigma_1 = 10\text{MPa} \quad \sigma_3 = -10\text{MPa} \quad \tau_{Max} = 10\text{MPa} \]

**Ductile**

Use either the Maximum Shear Stress Theory (MSST) or the Distortion Energy Theory (DET)

**MSST Theory**

\[ n = \frac{S_Y}{\sigma_1 - \sigma_3} = \frac{30}{10 - (-10)} = \frac{30}{20} = 1.5 \]

**DET Theory**

\[ \sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} = \sqrt{300} = 17.32\text{MPa} \]

\[ n = \frac{S_y}{\sigma_{VM}} = \frac{30\text{MPa}}{17.32\text{MPa}} = 1.73 \]
b. 0.2%C Carbon Steel

\[ S_Y = 65\text{Ksi} \quad \sigma_x = -5\text{Ksi} \quad \sigma_y = -35\text{Ksi} \quad \tau_{xy} = 10\text{Ksi} \]

In the plane XY the principal stresses are \(-1.973\text{Ksi}\) and \(-38.03\text{Ksi}\) with a maximum shear stress in the XY plane of \(18.03\text{Ksi}\).

In any orientation

\[ \sigma_1 = 0\text{Ksi} \quad \sigma_2 = -1.973\text{Ksi} \quad \sigma_3 = -38.03\text{Ksi} \]

\[ \tau_{\text{Max}} = 19.01\text{Ksi} \]

**Ductile**

**MSST Theory**

\[ n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{65}{0 - (-38.03)} = 1.71 \]

**DET Theory**

\[ \sigma_{VM} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3} = 38.03\text{Ksi} \]

\[ n = \frac{S_y}{\sigma_{VM}} = \frac{65\text{Ksi}}{38.03\text{MPa}} = 1.71 \]