Creating Estimated S-N Diagram

$S_f$ (or $S_e$) = $k_{surface}k_{size}k_{load}k_{temperature}k_{reliability}S'_f$ (or $S'_e$)

$S_f$ (or $S_e$) - corrected strength

$S'_f$ (or $S'_e$) - strength determined from standardized test

These equations are used to determine the corrected endurance limit $S_e$ or the corrected fatigue strength $S_f$ at a particular number of cycles $N$ in the high-cycle region of the S-N diagram. For the low-cycle region the following estimates are found:

- bending: $S_m = 0.9S_{ut}$
- axial loading: $S_m = 0.75S_{ut}$

$S_m$ material strength at $10^3$ cycles
The equation of the line from $S_m$ to $S_f$ (or $S_e$) is:

$$S_n = aN^b$$

for materials that possess an endurance limit the coefficients (a, b) can be calculated from the following two points:

- $S_n = S_m$ at $N = 10^3$
- $S_n = S_e$ at $N = 10^6$
for materials that do not possess an endurance limit, use:

Note: \( S_n = aN^b \) \hspace{1cm} \log(S_n) = \log a + b \log N

Boundary conditions:

Material with endurance limit

Material without endurance limit

\[ b = \frac{1}{z} \log \left( \frac{S_m}{S_e} \right) \]......where......\( z = \log N_1 - \log N_2 \)

Material with endurance limit

\( N_1 = 10^3 \) and \( N_2 = 10^6 \)

\[
\begin{array}{|c|c|}
\hline
N_2 (10^6) & z \\
\hline
1.0 & -3.000 \\
5.0 & -3.699 \\
10.0 & -4.000 \\
50.0 & -4.699 \\
100.0 & -5.000 \\
500.0 & -5.699 \\
\hline
\end{array}
\]
**Problem:** Create an estimated S-N diagram for a steel bar and define its equations. How many cycles of life can be expected if the alternating stress is **100MPa**.

**Given:** The $S_{ut}$ has been tested at **600MPa**. The bar is **150mm square** and has a hot rolled finish. The operating temperature is **500°C** maximum. The loading will be fully reverse bending.

**Assumptions:** Infinite life is required (ductile steel with endurance limit). A reliability factor of **99.9%** will be used.

**Solution:**

$$S_e' = 0.5S_{ut} = 300MPa$$

Loading factor is bending

$$k_{load} = 1.0$$

<table>
<thead>
<tr>
<th>Steels:</th>
<th>$S_e' \equiv 0.5S_{ut}$ for $S_{ut} &lt; 200$ ksi (1400 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_e' \equiv 100$ ksi (700 MPa) for $S_{ut} \geq 200$ ksi (1400 MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Irons:</th>
<th>$S_e' \equiv 0.4S_{ut}$ for $S_{ut} &lt; 60$ ksi (400 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_e' \equiv 24$ ksi (160 MPa) for $S_{ut} \geq 60$ ksi (400 MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aluminums:</th>
<th>$S_{f@5E8} \equiv 0.4S_{ut}$ for $S_{ut} &lt; 48$ ksi (330 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{f@5E8} \equiv 19$ ksi (130 MPa) for $S_{ut} \geq 48$ ksi (330 MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Copper Alloys:</th>
<th>$S_{f@5E8} \equiv 0.4S_{ut}$ for $S_{ut} &lt; 40$ ksi (280 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{f@5E8} \equiv 14$ ksi (100 MPa) for $S_{ut} \geq 40$ ksi (280 MPa)</td>
</tr>
</tbody>
</table>
The part is not round: \[ A_{95} = 0.05bh = 0.05 \times 150mm \times 150mm = 1125mm^2 \]

\[ d_{\text{equiv}} = \sqrt[2]{\frac{A_{95}}{0.0766}} = \sqrt[2]{\frac{1125mm^2}{0.0766}} = 121.2mm \]

The size factor:

\[ k_{\text{size}} = 1.189d_{\text{equivalent}}^{-0.097} \]

Surface Factor:

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>MPa</th>
<th>kpsi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>b</td>
</tr>
<tr>
<td>Ground</td>
<td>1.58</td>
<td>-0.085</td>
</tr>
<tr>
<td>Machined or cold-rolled</td>
<td>4.51</td>
<td>-0.265</td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>57.7</td>
<td>-0.718</td>
</tr>
<tr>
<td>As-forged</td>
<td>272</td>
<td>-0.995</td>
</tr>
</tbody>
</table>

\[ k_{\text{Surface}} = A \times S_{ut}^b = 57.7 \times (600)^{-0.718} = 0.584 \]
Temperature Factor: 

\[ k_{Temp} = 1 - 0.0058(T - 450) = 1 - 0.0058(500 - 450) = 0.71 \]

Reliability Factor

\[ k_{reliability} = 0.753 \]

<table>
<thead>
<tr>
<th>Reliability, %</th>
<th>Transformation Variate ( z_a )</th>
<th>Reliability Factor ( k_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>90</td>
<td>1.288</td>
<td>0.897</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
<td>0.868</td>
</tr>
<tr>
<td>99</td>
<td>2.326</td>
<td>0.814</td>
</tr>
<tr>
<td>99.9</td>
<td>3.091</td>
<td>0.753</td>
</tr>
<tr>
<td>99.99</td>
<td>3.719</td>
<td>0.702</td>
</tr>
<tr>
<td>99.999</td>
<td>4.265</td>
<td>0.659</td>
</tr>
<tr>
<td>99.9999</td>
<td>4.753</td>
<td>0.620</td>
</tr>
</tbody>
</table>

Corrected Endurance limit:

\[ S_e = k_{load} \times k_{size} \times k_{surf} \times k_{Temp} \times k_{reliability} \times S_e' \]

\[ S_e = 1.0 \times 0.747 \times 0.584 \times 0.71 \times 0.753 \times 300 = 70 MPa \]
Creating the S-N diagram

\[ S_n = aN^b \]

For bending:

\[ S_m = 0.9S_{ut} = 0.9 \times 600 = 540\text{MPa} \]

\[ b = \frac{1}{z} \log\left(\frac{S_m}{S_e}\right) = -\frac{1}{3} \log\left(\frac{540}{70}\right) = -0.295765 \]

\[ \log(a) = \log(S_m) - 3b = \log(540) - 3 \times (-0.295765) \]

\[ a = 4165.7 \]

\[ 100\text{MPa} = (4165.7) \times N^{-0.295765} \]

\[ N = 3.0 \times 10^5 \text{cycles} \]

\[
\begin{array}{|c|c|}
\hline
N_2 (10^6) & z \\ \hline
1.0 & -3.000 \\ \hline
5.0 & -3.699 \\ \hline
10.0 & -4.000 \\ \hline
50.0 & -4.699 \\ \hline
100.0 & -5.000 \\ \hline
500.0 & -5.699 \\ \hline
\end{array}
\]

\[ 10^5 \text{cycles} = 1 \times 10^5 \text{cycles} = 1 \times 10^5 \]

\[ 10^6 \text{cycles} = 1 \times 10^6 \text{cycles} = 1 \times 10^6 \]

\[ 10^7 \text{cycles} = 1 \times 10^7 \text{cycles} = 1 \times 10^7 \]

\[ 10^8 \text{cycles} = 1 \times 10^8 \text{cycles} = 1 \times 10^8 \]

\[ 10^9 \text{cycles} = 1 \times 10^9 \text{cycles} = 1 \times 10^9 \]

**Figure 6-34**

S-N Diagram and Alternating Stress Line Showing Failure Point for Example 6-1
Notches and Stress Concentrations

$K_t$ is the theoretical stress concentration factor. Notch sensitivity — $q$. Materials have different sensitivity to stress concentrations, referred to as the notch sensitivity of the material. In general, the more ductile a material is, the less sensitivity to notches it is. It depends on notch radius, the smaller the radius, the less sensitive the material is.

Neuber, Kuhn and Peterson have developed an approach to notch sensitivity Peterson’s equation

$$K_f = 1 + q(K_t - 1)$$

$$K_{fs} = 1 + q_{shear}(K_{ts} - 1)$$

where $a$ is Neuber’s constant and it is solely a function of the material, and $r$ is the notch radius (both expressed in inches).
### Table 6-6
Neuber's Constant for Steels

<table>
<thead>
<tr>
<th>$S_{ut}$ (ksi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.130</td>
</tr>
<tr>
<td>55</td>
<td>0.118</td>
</tr>
<tr>
<td>60</td>
<td>0.108</td>
</tr>
<tr>
<td>70</td>
<td>0.093</td>
</tr>
<tr>
<td>80</td>
<td>0.080</td>
</tr>
<tr>
<td>90</td>
<td>0.070</td>
</tr>
<tr>
<td>100</td>
<td>0.062</td>
</tr>
<tr>
<td>110</td>
<td>0.055</td>
</tr>
<tr>
<td>120</td>
<td>0.049</td>
</tr>
<tr>
<td>130</td>
<td>0.044</td>
</tr>
<tr>
<td>140</td>
<td>0.039</td>
</tr>
<tr>
<td>160</td>
<td>0.031</td>
</tr>
<tr>
<td>180</td>
<td>0.024</td>
</tr>
<tr>
<td>200</td>
<td>0.018</td>
</tr>
<tr>
<td>220</td>
<td>0.013</td>
</tr>
<tr>
<td>240</td>
<td>0.009</td>
</tr>
</tbody>
</table>

### Table 6-7
Neuber's Constant for Annealed Aluminum

<table>
<thead>
<tr>
<th>$S_{ut}$ (kpsi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.500</td>
</tr>
<tr>
<td>15</td>
<td>0.341</td>
</tr>
<tr>
<td>20</td>
<td>0.264</td>
</tr>
<tr>
<td>25</td>
<td>0.217</td>
</tr>
<tr>
<td>30</td>
<td>0.180</td>
</tr>
<tr>
<td>35</td>
<td>0.152</td>
</tr>
<tr>
<td>40</td>
<td>0.126</td>
</tr>
<tr>
<td>45</td>
<td>0.111</td>
</tr>
</tbody>
</table>

### Table 6-8
Neuber's Constant for Hardened Aluminum

<table>
<thead>
<tr>
<th>$S_{ut}$ (kpsi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.475</td>
</tr>
<tr>
<td>20</td>
<td>0.380</td>
</tr>
<tr>
<td>30</td>
<td>0.278</td>
</tr>
<tr>
<td>40</td>
<td>0.219</td>
</tr>
<tr>
<td>50</td>
<td>0.186</td>
</tr>
<tr>
<td>60</td>
<td>0.162</td>
</tr>
<tr>
<td>70</td>
<td>0.144</td>
</tr>
<tr>
<td>80</td>
<td>0.131</td>
</tr>
<tr>
<td>90</td>
<td>0.122</td>
</tr>
</tbody>
</table>
FIGURE 6-36 Part 1
Notch-Sensitivity Curves for Steels Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in Metal Fatigue by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

Note:
For torsional loading, use a curve for an $S_{ut}$ that is 20 ksi higher than that of the material selected.
**Problem:** A rectangular step steel bar (as shown below) is to be loaded in bending. Determine the stress-concentration factor for the given dimensions.

**Given:** $H=2\text{in}$, $h=1.8\text{in}$, $r=0.25\text{in}$. The material has $S_{ut}=100\text{ksi}$.

**Solution:** $K_t=1.56$

$$
\sqrt{a} = 0.062 \\
q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.062}{\sqrt{0.25}}} = 0.89
$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.89(1.56 - 1) = 1.50$$
**Design of a Cantilever Bracket for Fully Reversed Bending**

**Problem:** Design a cantilever bracket to support a fully reversed bending load of 500-lb amplitude for $10^9$ cycles with no failure. Its dynamic deflection can not exceed 0.01 in. Calculate the safety factor.

**Given:** Beam width ($b$) = 2 in; beam depth over length ($d$) = 1 in; beam depth in wall ($D$) = 1.125 in; fillet radius ($r$) = 0.3 in; applied load amplitude at point ($F$) = 500-lb; beam length ($l$) = 6 in; distance to load ($a$) = 5 in; distance for deflection calculation ($lx$) = 6 in; Modulus of elasticity ($E$) = $3 \times 10^7$ psi; ultimate tensile strength ($S_{ut}$) = 80 ksi for a steel. The cantilever has been machined and operates at a temperature of 120°F.
Solution:

\[ F_{\text{amplitude}} = 500\text{lb} \]
\[ F_{\text{mean}} = 0\text{lb} \]

\[ R = F_a = 500\text{lb} \]
\[ M = F_a \times a = 2500\text{lb \cdot in} \]

\[ I = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = 0.1667 \]
\[ c = \frac{d}{2} = \frac{1}{2} = 0.5 \]

Bending at the root

\[ \sigma_{a,nom} = \frac{Mc}{I} = \frac{2500 \times 0.5}{0.1667} = 7500 \text{ psi} \]

\[ \frac{D}{d} = \frac{1.125}{1} = 1.125 \]
\[ \frac{r}{d} = \frac{0.3}{1} = 0.3 \]

\[ K_t = 1.33 \]
The stress is a principal stress as there is no shear stress at the top surface and there are no other stresses present.

\[ \sigma_a = K_f \sigma_{a,nom} = 1.29 \times 7500 = 9675 \text{ psi} \]

\[ \tau_{ab} = \frac{\sigma_a}{2} = \frac{9675}{2} = 4832 \text{ psi} \]

\[ \sigma_{1a}, \sigma_{3a} = 9675 \text{ psi}, 0 \text{ psi} \]

\[ \sigma_{VM,a} = 9675 \text{ psi} \]
Creating the S-N diagram.

| Steels:          | \[ S_{e'} \equiv 0.5 S_{ut} \quad \text{for } S_{ut} < 200 \text{ ksi (1400 MPa)} \] |
|                 | \[ S_{e'} \equiv 100 \text{ ksi (700 MPa)} \quad \text{for } S_{ut} \geq 200 \text{ ksi (1400 MPa)} \] |
| Irons:          | \[ S_{e'} \equiv 0.4 S_{ut} \quad \text{for } S_{ut} < 60 \text{ ksi (400 MPa)} \] |
|                 | \[ S_{e'} \equiv 24 \text{ ksi (160 MPa)} \quad \text{for } S_{ut} \geq 60 \text{ ksi (400 MPa)} \] |
| Aluminums:      | \[ S_{f'@5E8} \equiv 0.4 S_{ut} \quad \text{for } S_{ut} < 48 \text{ ksi (330 MPa)} \] |
|                 | \[ S_{f'@5E8} \equiv 19 \text{ ksi (130 MPa)} \quad \text{for } S_{ut} \geq 48 \text{ ksi (330 MPa)} \] |
| Copper Alloys:  | \[ S_{f'@5E8} \equiv 0.4 S_{ut} \quad \text{for } S_{ut} < 40 \text{ ksi (280 MPa)} \] |
|                 | \[ S_{f'@5E8} \equiv 14 \text{ ksi (100 MPa)} \quad \text{for } S_{ut} \geq 40 \text{ ksi (280 MPa)} \] |

\[ S_{e'} = 0.5S_{ut} = 0.5 \times 80000 = 40000 \text{ psi} \]
\[ A_{95} = 0.05db = 0.05 \times 1 \times 2 = 0.1 \text{in}^2 \]

\[ d_{\text{equivalent}} = \sqrt{\frac{A_{95}}{0.0766}} = 1.14 \text{in} \]
Size factor: \[ A_{95} = 0.05db = 0.05 \times 1 \times 2 = 0.1 \text{in}^2 \]

\[ d_{\text{equivalent}} = \sqrt{\frac{A_{95}}{0.0766}} = 1.14 \text{in} \]

\[ \text{for } 0.3 \text{in} \leq d \leq 10. \text{in} : \ldots \ldots k_{\text{size}} = 0.869d^{-0.097} \]

\[ k_{\text{size}} = 0.869 \times (1.14)^{-0.097} = 0.86 \]

Load Factor:

\begin{itemize}
  \item \text{bending: } k_{\text{load}} = 1
  \item \text{axial: } k_{\text{load}} = 0.85
  \item \text{torsion: } k_{\text{load}} = 0.59
\end{itemize}

Pure bending \[ k_{\text{load}} = 1 \]

Surface factor:

\[ k_{\text{surface}} = aS_{ut}^b \]

\begin{table}
\begin{tabular}{lccccc}
\hline
\text{Surface Finish} & \text{MPa} & \text{kpsi} \\
\hline
Ground & 1.58 & 1.34 & -0.085 & -0.085 \\
Machined or cold-rolled & 4.51 & 2.7 & -0.265 & -0.265 \\
Hot-rolled & 57.7 & 14.4 & -0.718 & -0.718 \\
As-forged & 272 & 39.9 & -0.995 & -0.995 \\
\hline
\end{tabular}
\end{table}

\[ k_{\text{surf}} = AS_{ut}^b = 2.7 \times (80)^{-0.265} = 0.845 \]
Temperature factor: \[ k_{\text{Temp}} = 1.0 \]

Reliability factor: As only ten of these parts are required, a high % reliability is chosen (99.9%)
\[ k_{\text{reliability}} = 0.753 \]

Corrected endurance limit:
\[ S_e = k_{\text{load}} k_{\text{size}} k_{\text{surf}} k_{\text{Temp}} k_{\text{reliability}} S'_e \]
\[ S_e = 1 \times 0.86 \times 0.845 \times 1 \times 0.753 \times 40000 \text{ psi} \]
\[ S_e = 21843 \text{ psi} \]

Predicted Safety factor:
\[ n_{sf} = \frac{S_e}{\sigma_{VM,a}} = \frac{21843}{9675} = 2.26 \]

The beam deflection \( y \) is calculated
\[ y = \frac{F}{6EI} \left[ x^3 - 3ax^2 - (x - a)^3 \right] = \frac{500}{6(3 \times 10^7)(0.1667)} \left[ 6^3 - 3 \times 5 \times 6^2 - (6 - 5)^3 \right] = -0.005 \text{ in} \]
**Designing for fluctuating Uniaxial Stresses**

Many repeating and fluctuating stresses have non-zero mean components and these must be taken into account.

- **Modified-Goodman line** (for design)

\[
\sigma_a = S_e \left(1 - \frac{\sigma_m}{S_{ut}}\right)
\]

- **Gerber Parabola** (for analysis of failed parts)

\[
\sigma_a = S_e \left(1 - \frac{\sigma_m^2}{S_{ut}^2}\right)
\]
Design of a Cantilever Bracket for Fluctuating Bending

Problem: Design a cantilever bracket to support a fluctuating bending load of 100 to 1100-lb amplitude for $10^9$ cycles with no failure. Its dynamic deflection can not exceed 0.02in.. Calculate the safety factor.

Given: Beam width ($b$) = 2in; beam depth over length ($d$) = 1in; beam depth in wall ($D$) = 1.125in; fillet radius ($r$) = 0.3in; maximum applied load amplitude at point ($F$) = 1100-lb; minimum applied load amplitude at point ($F$) = 100-lb; beam length ($l$) = 6in; distance to load ($a$) = 5in; distance for deflection calculation ($lx$) = 6in; Modulus of elasticity ($E$) = $3 \times 10^7$psi; ultimate tensile strength ($S_{ut}$) = 80kpsi for a steel; yield strength ($S_y$) = 60kpsi. The cantilever has been machined and operates at a temperature of 120oF.
Solution:

\[ F_m = \frac{F_{Max} + F_{Min}}{2} = \frac{1100 + 100}{2} = 600 \text{ lb} \]

\[ F_a = \frac{F_{Max} - F_{Min}}{2} = \frac{1100 - 100}{2} = 500 \text{ lb} \]

\[ R_a = F_a = 500 \text{ lb} \ldots \ldots R_m = F_m = 600 \text{ lb} \ldots \ldots R_{Max} = F_{Max} = 1100 \text{ lb} \]

\[ M_a = F_a a = 500 \text{ lb} \times 5 \text{ in} = 2500 \text{ lb} - \text{ in} \]

\[ M_m = F_m a = 600 \text{ lb} \times 5 \text{ in} = 3000 \text{ lb} - \text{ in} \]

\[ M_{Max} = F_{Max} a = 1100 \text{ lb} \times 5 \text{ in} = 5500 \text{ lb} - \text{ in} \]

\[ I = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = 0.1667 \]

\[ c = \frac{d}{2} = \frac{1}{2} = 0.5 \]

\[ D = \frac{1.125}{d} = 1.125 \]

\[ r = \frac{0.3}{d} = 0.3 \]

\[ \sigma_{a,\text{nom}} = \frac{M_a c}{I} = \frac{2500 \times 0.5}{0.1667} = 7500 \text{ psi} \]

\[ \sigma_{m,\text{nom}} = \frac{M_m c}{I} = \frac{3000 \times 0.5}{0.1667} = 9000 \text{ psi} \]

\[ K_I = 1.33 \]
\[ \sqrt{a} = 0.080 \]
\[ q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.080}{\sqrt{0.30}}} = 0.87 \]

\[
K_f = 1 + q(K_t - 1) \\
K_f = 1 + 0.87(1.33 - 1) \\
K_f = 1.29
\]

Verify that the stress concentration are below yield strength of the material.

\[ \text{if } K_f |\sigma_{Max}| < S_y \text{ ....then..... } K_{fm} = K_f \]

\[
K_f |\sigma_{Max}| = K_f \left| \frac{M_{Max}c}{I} \right| = 1.29 \left| \frac{5500 \times 0.5}{0.1667} \right| = 21281 psi < S_y = 60000 psi
\]

\[
\sigma_a = K_f \sigma_{a,nom} = 1.29 \times 7500 psi = 9675 psi
\]

\[
\sigma_m = K_{f,m} \sigma_{m,nom} = 1.29 \times 9000 psi = 11610 psi
\]
Calculate the Von Mises for the alternating stress and the mean stress

\[
\sigma'_a = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \sigma_{y,a} + 3 \tau_{xy,a}^2} = \sqrt{9675^2 + 0^2 + 9675 \times 0 + 3 \times 0^2} = 9675 \text{ psi}
\]

\[
\sigma'_m = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m} \sigma_{y,m} + 3 \tau_{xy,m}^2} = \sqrt{11610^2 + 0^2 + 11610 \times 0 + 3 \times 0^2} = 11610 \text{ psi}
\]

Generating a S-N : Size factor:

\[
A_{95} = 0.05db = 0.05 \times 1 \times 2 = 0.1 \text{in}^2
\]

\[
d_{\text{equivalent}} = \sqrt{\frac{A_{95}}{0.0766}} = 1.14 \text{in}
\]

Load Factor:

Pure bending \( k_{\text{load}} = 1 \)

Surface factor:

\( k_{\text{surf}} = \frac{AS_{ut}^b}{2.7 \times (80)^{-0.265}} = 0.845 \)

Temperature factor:

\( k_{\text{Temp}} = 1.0 \)

Reliability factor: As only ten of these parts are required, a high % reliability is chosen (99.9%)
Corrected endurance limit:

\[ S'_e = k_{load} k_{size} k_{surf} k_{Temp} k_{reliability} S_e \]
\[ S'_e = 1 \times 0.86 \times 0.845 \times 1 \times 0.753 \times 40000 \text{ psi} \]
\[ S'_e = 21843 \text{ psi} \]

Various safety factors can be calculated:

\[ n_f = \frac{1}{\sigma_a + \sigma_m} \]
\[ n_f = \frac{1}{S_e / S_{ut}} \]

**Modified-Goodman:**
\[ n_f = \frac{1}{9675 + \frac{11610}{21883} + \frac{80000}{80000}} = 1.70 \]

**Langer (yielding):**
\[ n_y = \frac{S_y}{\sigma_a + \sigma_m} \]
\[ n_y = \frac{60000}{9675 + 11610} = 2.82 \]

The beam deflection \( y \) is calculated

\[ y = \frac{F_{Max}}{6EI} \left[ x^3 - 3ax^2 - (x - a)^3 \right] = \frac{1100}{6(3 \times 10^7)(0.1667)} \left[ 6^3 - 3 \times 5 \times 6^2 - (6 - 5)^3 \right] = -0.012 \text{ in} \]
Applying Stress-Concentration Effects with Fluctuating Stresses

\[ \text{if } K_f \sigma_{\text{Max}} < S_y \text{ then } K_{fm} = K_f \]

\[ \text{if } K_f \left| \sigma_{\text{Max}} \right| > S_y \text{ then } K_{fm} = \frac{S_y - K_f \sigma_{a,\text{nom}}}{\left| \sigma_{m,\text{nom}} \right|} \]

\[ \text{if } K_f \left| \sigma_{\text{Max, nom}} - \sigma_{\text{Min, nom}} \right| > 2S_y \text{ then } K_{fm} = 0 \]
Multiaxial Fluctuating Stresses

Problem: Determine the safety factors for the brackets tube shown.

Given:
Material: 2024-T4 aluminum with $S_y = 47 ksi$ and $S_{ut} = 68 ksi$.
Tube length $(l) = 6$ in.; arm $(a) = 8$ in.;
Tube outside diameter $(OD) = 2$ in.; Tube inside diameter $(ID) = 1.5$ in.
The applied load varies sinusoidally from $F = 340 \text{ to } -200$ lb
Assume finite life design $6 \times 10^7$ cycles.
The component has been machined. It operates at RT and with 99.9% reliability.

Notch radius at the wall = 0.25 in
Stress concentration factor for bending = 1.7
For shear = 1.35
Solution:

Aluminum does not have an endurance limit. As $S_{ut} > 48kpsi$ then $S_f=19kpsi$ at $5\times10^8$ cycles

Calculation of the correction factors:

Load Factor: Bending $k_{load} = 1$

Size factor: $\text{for } 0.3\text{in} \leq d \leq 10\text{.in} : \text{.........} k_{size} = 0.869d^{-0.097}$

This value is used despite the fact that both bending and torsion are present.

$A_{95} = 0.010462d^2$

$d_{\text{equivalent}} = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.010462d^2}{0.0766}} = 0.74\text{in}$

$k_{size} = 0.869 \times (0.74\text{in})^{-0.097} = 0.895$
Surface factor: (machined) \[ k_{surf} = A S_{ut}^b = 2.7 \times (68)^{-0.265} = 0.883 \]

Temperature factor: (room temperature) \[ k_{Temp} = 1.0 \]

Reliability factor: As only ten of these parts are required, a high % reliability is chosen (99.9%) \[ k_{reliability} = 0.753 \]

Corrected endurance limit:

At $5 \times 10^8$ cycles

The problem calls for a life of $6 \times 10^7$ cycles, so the strength value at that life must be estimated.

\[ S_n = aN^b \]

\[ S_f = k_{load} k_{size} k_{surf} k_{Temp} k_{reliability} S_e' \]

\[ S_f = 1 \times 0.895 \times 0.883 \times 1 \times 0.753 \times 19000 \text{ psi} \]

\[ S_f = 11299 \text{ psi} \]

$b = \frac{1}{z} \log \left( \frac{S_m}{S_f} \right) = - \frac{1}{5.699} \log \left( \frac{61200}{11299} \right) = -0.1287$

\[ \log(a) = \log(S_m) - 3b = \log(61200) - 3 \times (-0.1287) \]

\[ a = 148929 \]

\[ S_n = (148929) \times N^{-0.1287} = (148929) \times (6 \times 10^7)^{-0.1287} \]

\[ S_n = 14846 \text{ psi} \]
Notch sensitivity (for hardened aluminum $S_{ut} = 68000 \text{psi}$)

$$\sqrt{a} = 0.148$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.148}{\sqrt{0.25}}} = 0.773$$

**Bending**

$$K_f = 1 + q(K_t - 1)$$

$$K_f = 1 + 0.773(1.7 - 1)$$

$$K_f = 1.541$$

**Torsion**

$$K_{f,s} = 1 + q(K_{t,s} - 1)$$

$$K_{f,s} = 1 + 0.773(1.35 - 1)$$

$$K_{f,s} = 1.270$$

### Table 6-7
Neuber's Constant for Annealed Aluminum

<table>
<thead>
<tr>
<th>$S_{ut}$ (ksi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.500</td>
</tr>
<tr>
<td>15</td>
<td>0.341</td>
</tr>
<tr>
<td>20</td>
<td>0.264</td>
</tr>
<tr>
<td>25</td>
<td>0.217</td>
</tr>
<tr>
<td>30</td>
<td>0.180</td>
</tr>
<tr>
<td>35</td>
<td>0.152</td>
</tr>
<tr>
<td>40</td>
<td>0.126</td>
</tr>
<tr>
<td>45</td>
<td>0.111</td>
</tr>
</tbody>
</table>

### Table 6-8
Neuber's Constant for Hardened Aluminum

<table>
<thead>
<tr>
<th>$S_{ut}$ (ksi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.475</td>
</tr>
<tr>
<td>20</td>
<td>0.380</td>
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<tr>
<td>30</td>
<td>0.278</td>
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<tr>
<td>40</td>
<td>0.219</td>
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<tr>
<td>50</td>
<td>0.186</td>
</tr>
<tr>
<td>60</td>
<td>0.162</td>
</tr>
<tr>
<td>70</td>
<td>0.144</td>
</tr>
<tr>
<td>80</td>
<td>0.131</td>
</tr>
<tr>
<td>90</td>
<td>0.122</td>
</tr>
</tbody>
</table>
The bracket tube is loaded in bending (as a cantilever) and in torsion. All the loading are maximum at the wall. Need to find the alternating and mean components of the applied force, moment and torque at the wall.

\[ F_a = \frac{F_{Max} - F_{Min}}{2} = \frac{340 - (-200)}{2} = 270\text{lb} \]

\[ F_m = \frac{F_{Max} + F_{Min}}{2} = \frac{340 + (-200)}{2} = 70\text{lb} \]

\[ M_a = F_a l = 270\text{lb} \times 6\text{in} = 1620\text{lb in} \]

\[ M_m = F_m l = 70\text{lb} \times 6\text{in} = 420\text{lb in} \]

\[ T_a = F_a a = 270\text{lb} \times 8\text{in} = 2160\text{lb in} \]

\[ T_m = F_m a = 70\text{lb} \times 8\text{in} = 560\text{lb in} \]
The fatigue stress concentration factor for the mean stresses depends on the relationship between the maximum local stress in the notch and the yield strength

\[ \text{if } K_f |\sigma_{Max}| < S_y \text{ then } K_{fm} = K_f \]

\[ K_f |\sigma_{Max}| = K_f \left| \frac{F_{Max} l c}{l} \right| = 1.541 \left| \frac{340 \times 6 \times 1}{0.5369} \right| = 5855 \text{ psi} < S_y = 47000 \text{ psi} \]

bending \( K_{fm} = K_f = 1.541 \)

torsion \( K_{fm} = K_f = 1.270 \)

The largest tensile bending stress will be at the top or bottom of the cantilever. The largest torsional shear stress will be all around the outer circumference of the tube.
Finding the amplitude and mean components due to bending and torsion.

$$\sigma_a = K_f \frac{M_a c}{I}$$
$$\tau_{a,torsion} = K_{F, shear} \frac{T_a r}{J}$$
$$\sigma_m = K_{fm} \frac{M_m c}{I}$$
$$\tau_{m,torsion} = K_{F, shear, mean} \frac{T_m r}{J}$$

Finding the alternating/amplitude and mean Von Mises stresses at $A$.

$$\sigma'_a = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \times \sigma_{y,a} + 3\tau_{xy,a}^2}$$
$$\sigma'_m = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m} \times \sigma_{y,m} + 3\tau_{xy,m}^2}$$
Various safety factors can be calculated:

It assumes that the alternate and mean component will have a constant ratio.

Modified-Goodman:

\[ n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} \]

Langer (yielding):

\[ n_y = \frac{S_y}{\sigma_a + \sigma_m} \]

\[ n_f = \frac{1}{\frac{6419}{14846} + \frac{1664}{68000}} = 2.2 \]

\[ n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{47000}{6419 + 1664} = 5.82 \]

We need to check the shear due to transverse loading at point B in the neutral axis where the torsional shear is also maximal.

\[ \tau_{a,\text{transverse}} = K_{f,\text{shear}} \frac{2V_a}{A} = 1.270 \frac{2 \times 270}{1.374} = 499 \text{ psi} \]

\[ \tau = \frac{VQ}{Ib} \]

\[ \tau_{m,\text{transverse}} = K_{f,\text{shear,mean}} \frac{2V_m}{A} = 1.270 \frac{2 \times 70}{1.374} = 129 \text{ psi} \]

\[ \tau_{a,\text{total}} = \tau_{a,\text{transverse}} + \tau_{a,\text{torsion}} = 499 + 2556 = 3055 \text{ psi} \]

\[ \tau_{m,\text{total}} = \tau_{m,\text{transverse}} + \tau_{m,\text{torsion}} = 129 + 663 = 729 \text{ psi} \]
\[
\sigma'_a = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \times \sigma_{y,a} + 3\tau_{xy,a}^2}
\]
\[
\sigma'_a = \sqrt{0^2 + 0^2 - 0 \times 0 + 3 \times 3055^2} = 5291 \text{ psi}
\]
\[
\sigma'_m = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m} \times \sigma_{y,m} + 3\tau_{xy,m}^2}
\]
\[
\sigma'_m = \sqrt{0^2 + 0^2 - 0 \times 0 + 3 \times 792^2} = 1372 \text{ psi}
\]

Safety factors:

\[
\text{Modified-Goodman: } n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}
\]

\[
\text{Langer (yielding): } n_y = \frac{S_y}{\sigma_a + \sigma_m}
\]

It assumes that the alternate and mean component will have a constant ratio.

\[
n_f = \frac{1}{\frac{5291}{14846} + \frac{1372}{68000}} = 2.7
\]

\[
n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{47000}{5291+1372} = 7.05
\]

Both points A and B are safe against fatigue failure.
We have a rotating shaft, as shown, made of AISI 1040 hot rolled ($S_y = 290\,\text{MPa}$; $S_{ut} = 524\,\text{MPa}$; $E = 210\,\text{GPa}$). Determine the Moment it can support for 100,000 cycle life.

\[
\frac{D}{d} = \frac{6}{4} = 1.5
\]
\[
\frac{r}{d} = \frac{0.2}{4} = 0.05
\]

\[K_t = 2.03\]
### Surface Finish

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>A</th>
<th>b</th>
<th>A</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>1.58</td>
<td>-0.085</td>
<td>1.34</td>
<td>-0.085</td>
</tr>
<tr>
<td>Machined or cold rolled</td>
<td>4.51</td>
<td>-0.265</td>
<td>2.7</td>
<td>-0.265</td>
</tr>
<tr>
<td>Hot-rolled</td>
<td>57.7</td>
<td>-0.718</td>
<td>14.4</td>
<td>-0.718</td>
</tr>
<tr>
<td>As-forged</td>
<td>272</td>
<td>-0.995</td>
<td>39.9</td>
<td>-0.995</td>
</tr>
</tbody>
</table>

### k_{surface} = aS_{ut}^b

**for** $d \leq 0.3\text{in (8mm)} : \ldots k_{size} = 1$  
**for** $0.3\text{in} \leq d \leq 10\text{in} : \ldots k_{size} = 0.869d^{-0.097}$  
**for** $8\text{mm} \leq d \leq 250\text{mm} : \ldots k_{size} = 1.189d^{-0.097}$

### $A_{95\sigma} = \pi \left( \frac{d^2}{4} - \frac{(0.95d)^2}{4} \right) = 0.0766d^2$

$$d_{\text{equivalent}} = \sqrt[4]{\frac{A_{95\sigma}}{0.0766}}$$

### Reliability, %

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>$S_i/S_{ut}$</th>
<th>Temperature, °F</th>
<th>$S_i/S_{ut}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.000</td>
<td>70</td>
<td>1.000</td>
</tr>
<tr>
<td>50</td>
<td>1.010</td>
<td>100</td>
<td>1.008</td>
</tr>
<tr>
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<td>200</td>
<td>1.020</td>
</tr>
<tr>
<td>120</td>
<td>1.023</td>
<td>300</td>
<td>1.024</td>
</tr>
<tr>
<td>200</td>
<td>1.020</td>
<td>400</td>
<td>1.018</td>
</tr>
<tr>
<td>250</td>
<td>1.000</td>
<td>500</td>
<td>0.995</td>
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<tr>
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<td>0.978</td>
<td>600</td>
<td>0.943</td>
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<td>500</td>
<td>0.764</td>
<td>1000</td>
<td>0.709</td>
</tr>
<tr>
<td>550</td>
<td>0.672</td>
<td>1100</td>
<td>0.586</td>
</tr>
<tr>
<td>600</td>
<td>0.549</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Reference by 3-8.*
Determine the Fatigue Stress Concentration

\[
\sqrt{a} - 0.080 = \frac{0.093 - 0.080}{76 - 80}
\]

\[
\sqrt{a} = \left( \frac{0.093 - 0.080}{70 - 80} \right) (76 - 80) + 0.080
\]

\[
\sqrt{a} = 0.085
\]

\[
\sqrt{a} = 0.085
\]

\[
q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.085}{\sqrt{2}/25.4}} = 0.768
\]

\[
K_f = 1 + q(K_t - 1)
\]

\[
K_f = 1 + 0.768 \times (2.03 - 1)
\]

\[
K_f = 1.791
\]

TS=524MPa=76ksi

<table>
<thead>
<tr>
<th>$S_{ut}$ (ksi)</th>
<th>$\sqrt{a}$ (in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.130</td>
</tr>
<tr>
<td>55</td>
<td>0.118</td>
</tr>
<tr>
<td>60</td>
<td>0.108</td>
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<tr>
<td>70</td>
<td>0.093</td>
</tr>
<tr>
<td>80</td>
<td>0.080</td>
</tr>
<tr>
<td>90</td>
<td>0.070</td>
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<tr>
<td>100</td>
<td>0.062</td>
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<td>110</td>
<td>0.055</td>
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<td>120</td>
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<tr>
<td>130</td>
<td>0.044</td>
</tr>
<tr>
<td>140</td>
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</tr>
<tr>
<td>160</td>
<td>0.031</td>
</tr>
<tr>
<td>180</td>
<td>0.024</td>
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<tr>
<td>200</td>
<td>0.018</td>
</tr>
<tr>
<td>220</td>
<td>0.013</td>
</tr>
<tr>
<td>240</td>
<td>0.009</td>
</tr>
</tbody>
</table>
Determine the S-N Diagram

\[ S_e' = 0.5 \times S_{UT} = 0.5 \times 524 \text{MPa} = 262 \text{MPa} \]

\[ k_{load-bending} = 1 \]

\[ A_{95} = 0.0766d^2 = 0.0766 \times 40^2 = 122.56 \text{mm}^2 \]

\[ d_{equivalent} = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{122.56}{0.0766}} = 40 \text{mm} \]

\[ k_{size} = 1.189 \times d^{-0.097} = 1.189 \times (40 \text{mm})^{-0.097} = 0.831 \]

\[ k_{surf} = A_S^b \times S_{ut} = 57.7 \times (524)^{-0.718} = 0.6437 \]

\[ k_{Temp} = 1.0 \]

\[ k_{reliability} = 0.753 \]

\[ S_f = S_e' \times k_{load} \times k_{size} \times k_{surf} \times k_{Temp} \times k_{reliability} \]

\[ S_f = 262 \times 1 \times 0.831 \times 0.6437 \times 1 \times 0.753 \]

\[ S_f = 105.5 \text{MPa} \]
$S_n = aN^b$

$b = \frac{1}{z} \log \left( \frac{S_m}{S_e} \right) = -\frac{1}{3} \log \left( \frac{471.6}{105.5} \right) = -0.21677$

$\log(S_n) = \log a + b \log N$

$\log a = \log(S_m) - 3b = \log(471.6) - 3 \times (-0.21677)$

$a = 2108$

$\log(S_f) = b \times \log(N) + \log(a)$

$\log(S_f) = \frac{\log(471.6) - \log(105.5)}{\log(10^3) - \log(10^6)} \times \log(10^5) + \log(2108)$

$log(S_f) = 2.24$

$S_f = 173.8MPa$

<table>
<thead>
<tr>
<th>$N_2(10^6)$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-3.000</td>
</tr>
<tr>
<td>5.0</td>
<td>-3.699</td>
</tr>
<tr>
<td>10.0</td>
<td>-4.000</td>
</tr>
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<td>-4.699</td>
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<td>100.0</td>
<td>-5.000</td>
</tr>
<tr>
<td>500.0</td>
<td>-5.699</td>
</tr>
</tbody>
</table>
Calculate the stress (amplitude and mean)

\[ I = \frac{\pi r^4}{4} = \frac{\pi \times (0.02^4)}{4} = 0.1257 \times 10^{-6} \]

\[ \sigma_{\text{Max}} = \frac{M_y}{I} = \frac{M \times 0.02}{0.1257 \times 10^{-6}} = 1.5915 \times 10^5 \times M \]

\[ \sigma_{\text{Min}} = -\frac{M_y}{I} = -\frac{M \times 0.02}{0.1257 \times 10^{-6}} = -1.5915 \times 10^5 \times M \]

\[ \sigma_{\text{Mean}} = 0 \]

\[ \sigma_{\text{Amplitude}} = 1.5915 \times 10^5 \times M \]

\[ \sigma_{\text{Amplitude-Corrected}} = K_f \times \sigma_{\text{Amplitude}} = 1.791 \times 1.5915 \times 10^5 \times M \]

\[ \sigma_{\text{Amplitude-Corrected}} = 0.28504 \times 10^6 \times M \]

\[ \sigma_{\text{Amplitude-Corrected}} = S_f = 173.8 \text{MPa} \]

\[ \text{Moment} = 609.7N - m \]
**Problem:** Determine the safety factor for the component shown below. Consider all stress concentration factors at the wall equals to 1.0 and all notch radius of 0.25in. At 1in from the wall the stress concentration factors are $K_{\text{bending}}=1.8; K_{\text{torsion}}=1.6; K_{\text{shear}}=1.3; K_{\text{axial}}=1.5$

**Given:**

- The load $P$ varies between $P_{\text{max}}=8000\text{lb}$ and $P_{\text{min}}=-4000\text{lb}$
- The load $R$ varies between $\pm 1000\text{lb}$
- AISI 4340 Oil Quenched at 855°C and Temper at 230°C for 4 hours
  - (Sy=200ksi; Sut=260ksi)
- Machined component working at room temperature and a reliability of 99.99%
Determine the S-N Diagram

\[ S_e' = 100\text{Ksi} \quad k_{\text{Temp}} = 1.0 \]

\[ k_{\text{load-bending}} = 1 \quad k_{\text{reliability}} = 0.702 \]

\[ k_{\text{surf}} = A S_{\text{ut}}^b = 2.7 \times (260)^{-0.265} = 0.619 \]

Determine the Fatigue Stress Concentration

\[ \sqrt{a} = 0.005 \]
\[ q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.005}{\sqrt{0.25}}} = 0.99 \]

\[ K_f = 1 + q(K_t - 1) \]
\[ K_{f,\text{bending}} = 1 + 0.986 \times (1.8 - 1) \quad K_{f,\text{bending}} = 1.788 \]
\[ K_{f,\text{axial}} = 1 + 0.986 \times (1.5 - 1) \quad K_{f,\text{axial}} = 1.493 \]
\[ K_{f,\text{shear}} = 1 + 0.986 \times (1.3 - 1) \quad K_{f,\text{shear}} = 1.296 \]

\[ A_{95} = 0.05 \times 1.5 \times 2.4 = 0.18\text{in}^2 \]
\[ d_{\text{equivalent}} = \sqrt{\frac{A_{95}}{0.0766}} = \sqrt{\frac{0.18}{0.0766}} = 1.53\text{in} \]
\[ k_{\text{size}} = 0.869 \times d^{-0.097} = 0.869 \times (1.53\text{mm})^{-0.097} = 0.833 \]

\[ S_f = S_e' \times k_{\text{load}} \times k_{\text{size}} \times k_{\text{surf}} \times k_{\text{Temp}} \times k_{\text{reliability}} \]
\[ S_f = 100 \times 1 \times 0.833 \times 0.619 \times 1 \times 0.702 \]
\[ S_f = 36.197\text{Ksi} \]

### Table 6-6
Neuber's Constant for Steels

<table>
<thead>
<tr>
<th>( S_{\text{ut}} ) (ksi)</th>
<th>( \sqrt{a} ) (in^{0.5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.130</td>
</tr>
<tr>
<td>55</td>
<td>0.118</td>
</tr>
<tr>
<td>60</td>
<td>0.108</td>
</tr>
<tr>
<td>70</td>
<td>0.093</td>
</tr>
<tr>
<td>80</td>
<td>0.080</td>
</tr>
<tr>
<td>90</td>
<td>0.070</td>
</tr>
<tr>
<td>100</td>
<td>0.062</td>
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<tr>
<td>110</td>
<td>0.055</td>
</tr>
<tr>
<td>120</td>
<td>0.049</td>
</tr>
<tr>
<td>130</td>
<td>0.044</td>
</tr>
<tr>
<td>140</td>
<td>0.039</td>
</tr>
<tr>
<td>160</td>
<td>0.031</td>
</tr>
<tr>
<td>180</td>
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</tr>
<tr>
<td>200</td>
<td>0.018</td>
</tr>
<tr>
<td>220</td>
<td>0.013</td>
</tr>
<tr>
<td>240</td>
<td>0.009</td>
</tr>
</tbody>
</table>
\[
\sqrt{\left(6^2 + 1.75^2\right)} = 6.25
\]

\[
Area = A = 1.5 \times 2.4 = 3.6in^2
\]

\[
I_x = \frac{1.5 \times 2.4^3}{12} = 1.728in^4
\]

\[
I_z = \frac{2.4 \times 1.5^3}{12} = 0.675in^4
\]

At the wall:

\[
M_x = P \times 5
\]

\[
M_z = R \times \left(\frac{1.75}{6.25}\right) \times 6 + R \times \left(\frac{6.00}{6.25}\right) \times 3.25
\]

\[
R_y = R \times \left(\frac{6.00}{6.25}\right)
\]

\[
V_x = R \times \left(\frac{1.75}{6.25}\right)
\]

\[
V_z = P
\]

\[
R = \frac{P}{25.6} + \frac{25.325}{75.1}
\]
Point A:

\[
\sigma_y = \frac{M_x \times \left(\frac{2.4}{2}\right)}{I_x} - \frac{R \times \left(\frac{6}{6.25}\right)}{A} = \frac{P \times 5 \times 1.2}{1.728} - 0.266R = 3.472P - 0.266R
\]

\[
\tau_{yx} = \frac{3 \times V_x}{2 \times A} = \frac{1.5 \times R \left(\frac{1.75}{6.25}\right)}{2 \times 3.6} = 0.0583R
\]

\[
P_{\text{Max}} = 8000\text{lb} \quad P_{\text{Min}} = -4000\text{lb}
\]

\[
P_{\text{Mean}} = 2000\text{lb} \quad P_{\text{Amplitude}} = 6000\text{lb}
\]

\[
\sigma_y = 3.472P - 0.266R
\]

\[
R_{\text{Max}} = 1000\text{lb} \quad R_{\text{Min}} = -1000\text{lb}
\]

\[
R_{\text{Mean}} = 0\text{lb} \quad R_{\text{Amplitude}} = 1000\text{lb}
\]

\[
\sigma_{y,\text{Mean}} = 3.472 \times 2000 = 6944\text{ psi}
\]

\[
\sigma_{y,\text{Amplitude}} = 3.472 \times 6000 - 0.266 \times 1000 = 20566\text{ psi}
\]

\[
\tau_{yx,\text{Mean}} = 0
\]

\[
\tau_{yx,\text{Amplitude}} = 0.0583 \times 1000 = 58.3\text{ psi}
\]
Finding the amplitude and mean Von Mises stresses at A.

\[
\sigma_a' = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \times \sigma_{y,a} + 3\tau_{xy,a}^2}
\]

\[
\sigma_a = \sqrt{0^2 + 20566^2 - 0 \times 20566 + 3 \times 58.3^2} = 20566 \text{ psi}
\]

\[
\sigma_m' = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m} \times \sigma_{y,m} + 3\tau_{xy,m}^2}
\]

\[
\sigma_m = \sqrt{0^2 + 6944^2 - 0 \times 6944 + 3 \times 0^2} = 6944 \text{ psi}
\]

Safety factors for Point A at the wall:

**Modified-Goodman:**

\[
n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{20566}{31197} + \frac{6944}{260000}} = 1.46
\]

**Langer (yielding):**

\[
n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{200000}{20566 + 6944} = 7.27
\]
**Point B:**

\[
\sigma_y = \frac{M_z \times \left(\frac{1.5}{2}\right)}{I_z} - \frac{R \times \left(\frac{6}{6.25}\right)}{A} = \frac{4.8 \times R \times 0.75}{0.675} - 0.2666R = 5.066R
\]

\[
\tau_{yz} = \frac{3 \times V_z}{2 \times A} = \frac{1.5 \times P}{2 \times 3.6} = 0.208P
\]

\[
P_{Max} = 8000\text{lb} \quad \quad P_{Min} = -4000\text{lb} \quad \quad R_{Max} = 1000\text{lb} \quad \quad R_{Min} = -1000\text{lb} \\
P_{Mean} = 2000\text{lb} \quad \quad P_{Amplitude} = 6000\text{lb} \quad \quad R_{Mean} = 0\text{lb} \quad \quad \text{R}_{Amplitude} = 1000\text{lb}
\]

\[
\tau_{yz,\text{Mean}} = 0.208 \times P = 416\text{ psi} \\
\sigma_{y,\text{Amplitude}} = 5.066 \times 1000 = 5066\text{ psi} \\
\tau_{yz,\text{Amplitude}} = 0.208 \times 6000 = 1248\text{ psi}
\]

\[
\sigma_a = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a} \times \sigma_{y,a} + 3\tau_{xy,a}^2}
\]

\[
\sigma_m = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m} \times \sigma_{y,m} + 3\tau_{xy,m}^2}
\]

\[
n_f = \frac{1}{\frac{5508}{31197} + \frac{720}{260000}} = 5.6
\]

\[
n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{200000}{5508 + 720} = 32.11
\]
At 1 in from the wall: \[ \sqrt{6^2 + 1.75^2} = 6.25 \]

\[ \text{Area} = A = 1.5 \times 2.4 = 3.6 \text{in}^2 \]

\[ I_x = \frac{1.5 \times 2.4^3}{12} = 1.728 \text{in}^4 \]

\[ I_z = \frac{2.4 \times 1.5^3}{12} = 0.675 \text{in}^4 \]

\[ M_z = R \times \left( \frac{1.75}{6.25} \right) \times 5 + R \times \left( \frac{6.00}{6.25} \right) \times 3.25 \]

\[ R_y = R \times \left( \frac{6.00}{6.25} \right) \]

\[ V_x = R \times \left( \frac{1.75}{6.25} \right) \]

\[ V_z = P \]

\[ M_x = P \times 4 \]
\[
\sigma_y = K_{f,bending} \times \frac{M_x \times \left( \frac{2.4}{2} \right)}{I_x} - K_{f,axial} \times \frac{R \times \left( \frac{6}{6.25} \right)}{A}
\]

\[
\sigma_y = K_{f,bending} \times \frac{P \times 4 \times 1.2}{1.728} - K_{f,axial} \times 0.2666R
\]

\[
\sigma_y = 1.7883 \times 2.78P - 1.493 \times 0.266R = 4.97P - 0.397R
\]

\[
\tau_{yx} = K_{f,shear} \times \frac{3 \times V_x}{2 \times A} = 1.296 \times \frac{1.5 \times R \left( \frac{1.75}{6.25} \right)}{2 \times 3.6} = 0.0755R
\]

\[
P_{Max} = 8000lb \ldots \ldots P_{Min} = -4000lb \\
R_{Max} = 1000lb \ldots \ldots R_{Min} = -1000lb
\]

\[
P_{Mean} = 2000lb \ldots \ldots P_{Amplitude} = 6000lb \\
R_{Mean} = 0lb \ldots \ldots R_{Amplitude} = 1000lb
\]

\[
\sigma_{y,Mean} = 4.97 \times 2000 = 9940psi \\
\sigma_{y,Amplitude} = 4.97 \times 6000 - 0.397 \times 1000 = 29423psi \\
\tau_{yx,Mean} = 0 \\
\tau_{yx,Amplitude} = 0.0755 \times 1000 = 75.5psi
\]
Finding the amplitude and mean Von Mises stresses at A.

\[ \sigma'_a = \sqrt{\sigma_{x,a}^2 + \sigma_{y,a}^2 - \sigma_{x,a}\times\sigma_{y,a} + 3\tau_{xy,a}^2} \]

\[ \sigma'_a = \sqrt{0^2 + 29423^2 - 0 \times 29423 + 3 \times 75.5^2} = 29423 \text{ psi} \]

\[ \sigma'_m = \sqrt{\sigma_{x,m}^2 + \sigma_{y,m}^2 - \sigma_{x,m}\times\sigma_{y,m} + 3\tau_{xy,m}^2} \]

\[ \sigma'_m = \sqrt{0^2 + 9940^2 - 0 \times 9940 + 3 \times 0^2} = 9940 \text{ psi} \]

Safety factors for Point A at 1 in from the wall:

Modified-Goodman: \[ n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} \]

\[ n_f = \frac{1}{\frac{29423}{31197} + \frac{9940}{260000}} = 1.02 \]

Langer (yielding): \[ n_y = \frac{S_y}{\sigma_a + \sigma_m} \]

\[ n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{200000}{29423 + 9940} = 5.08 \]
Examples of fatigue Failure

Failure of a steam turbine blade from a nuclear power plant due to fatigue
A fatigue crack that started at the site of a lightning strike is shown below.
A rotating shaft loaded by the 5kN and 10kN forces is represented below. The grinding relief groove at B is 2.5mm deep. The cylindrical surface AB is ground but the groove is machined. The shaft is made of a steel hardened and tempered to $S_{UT}=1300\text{MPa}$ and a $S_{Yield}=1000\text{MPa}$ according to the BS 826M40 (EN26). The shaft transmit a power of 15kW at 900RPM between the loads. A safety factor (on load) of 2 corresponding to a life of $0.35\times10^6$ cycles has been used for the shaft design. Is the design correct? If not determine the new life for a S.F.=2. The shaft operates at room temperature with a reliability of 99.9%.
$K_{\text{bending}} = 1.63 \ldots K_{\text{Torsion}} = 1.32$

Determine the Fatigue Stress Concentration

$K_f = 1 + q(K_t - 1)$

$K_{f,\text{bending}} = 1 + 0.954 \times (1.63 - 1) \ldots \ldots K_{f,\text{bending}} = 1.601$

$K_{f,\text{torsion}} = 1 + 0.954 \times (1.32 - 1) \ldots \ldots K_{f,\text{shear}} = 1.305$

$1300 \text{ MPa} = 188.5 \text{ ksi}$

\[\sqrt{a} = 0.0215\]

\[q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.0215}{\sqrt{5}/25.4}} = 0.954\]
Determine the S-N Diagram

\[ S_e = 650 \text{MPa} \]

\[ k_{\text{Temp}} = 1.0 \]

\[ k_{\text{load-bending}} = 1 \]

\[ k_{\text{surface}} = aS_{\text{ut}}^b \]

\[ k_{\text{reliability}} = 0.753 \]

\[ S_{\text{surf}} = AS_{\text{ut}}^b = 4.51 \times (1300)^{0.265} = 0.6745 \]

\[ A_{95} = 0.0766d^2 = 0.0766 \times 35^2 = 93.835 \text{mm}^2 \]

\[ d_{\text{equivalent}} = \sqrt[4]{\frac{A_{95}}{0.0766}} = \sqrt[4]{\frac{93.835}{0.0766}} = 35 \text{mm} \]

\[ k_{\text{size}} = 1.189 \times d^{-0.097} = 1.189 \times (35 \text{mm})^{-0.097} = 0.8422 \]

\[ S_f = S'_e \times k_{\text{load}} \times k_{\text{size}} \times k_{\text{surf}} \times k_{\text{Temp}} \times k_{\text{reliability}} \]

\[ S_f = 650 \times 1 \times 0.8422 \times 0.6745 \times 1 \times 0.753 \]

\[ S_f = 278 \text{MPa} \]

\[ 0.9 \times 1300 = 1170 \text{MPa} \]

\[ S_n = aN^b \]

\[ b = \frac{1}{3} \log \left( \frac{S_m}{S_e} \right) = -\frac{1}{3} \log \left( \frac{1170}{278} \right) = -0.208 \]

\[ \log(S_n) = \log a + b \log N \]

\[ \log a = \log(S_n) - b \log N \]

\[ \log(a) = \log(1170) - 3 \times (-0.208) \]

\[ \log(a) = 3.69218 \Rightarrow a = 4922.4 \]

\[ \log(S_f) = b \times \log(N) + \log(a) \]

\[ \log(S_f) = \frac{\log(1170) - \log(278)}{\log(10^3)} \times \log(3.5 \times 10^5) + \log(4922.4) \]

\[ \log(S_f) = 2.539 \]

\[ S_f = 345.95 \text{MPa} \]
\[
\sum M_A = 0 = 5 \times 100 + 250 \times 10 - R_2 \times 350 \Rightarrow R_2 = 8.571N - m = 6.428N - m
\]

\[
P = \frac{2\pi n T}{60} \; \text{....n(rpm)\cdot T(N - m)\cdot P(watts)}
\]

\[
T = \frac{60 \times 15000}{2\pi \times 900} = 159.15N - m
\]

**Critical point at B:**

\[
M_B = 749.9N \cdot m \quad T_B = 159.15N \cdot m
\]

**Bending moment gives the stress amplitude, while the Torque gives the mean stress.**

\[
\sigma_{bending,\; amplitude} = K_{bending} \times \frac{M \times c}{I}
\]

\[
\tau_{torque,\; mean} = K_{torque} \times \frac{T \times c}{J}
\]

\[
I = \frac{\pi}{64} (0.035)^4 = 7.366 \times 10^{-8} m^4
\]

\[
J = \frac{\pi}{32} (0.035)^4 = 14.73 \times 10^{-8} m^4
\]
Finding the Von Mises stresses

\[ \sigma_{VM,a} = 285.23 \text{MPa} \]
\[ \sigma_{VM,m} = \sqrt{3 \times (24.68 \text{MPa})^2} = 42.74 \text{MPa} \]

Calculating the new life

\[ \frac{1}{2} = \frac{285.23}{\sigma_N} + \frac{42.74}{1300} \Rightarrow \sigma_N = 574.2 \text{MPa} \]

Related formulas:

\[ \tau_{torque,mean} = K_{torque} \times \frac{T \times c}{J} \]
\[ \sigma_{bending,amplitude} = K_{bending} \times \frac{M \times c}{I} \]

Modified Goodman

\[ \frac{1}{S.F.} = \frac{285.23}{345.95} + \frac{42.74}{1300} \Rightarrow S.F. = 1.17 \]

Langer

\[ Langer SF = \frac{\sigma_{Yield}}{\sigma_{VM,mean} + \sigma_{VM,amp}} = \frac{1000 \text{MPa}}{285.23 \text{MPa} + 42.74 \text{MPa}} = 3.05 \]

\[ \log(N) = \log(S_f) - \log(a) \Rightarrow \log(N) = \frac{(\log(S_f) - \log(a))}{b} \]
\[ \log(N) = \frac{\log(574.2) - \log(4922.4)}{-0.208} = 4.486 \]
\[ N = 30620 \text{cycles} \]