

**INVERSE PROCEDURE FOR ESTIMATING VERTICALLY DISTRIBUTED SOIL
HYDRAULIC PARAMETERS USING GPR**

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ABSTRACT

Knowledge of the vertical distribution of the volumetric soil moisture content is essential for understanding the movement of water and its availability for plant uptake, or loss by deep percolation. A procedure is presented for estimating the depth-specific dielectric constant from bulk average dielectric constants obtained with surface ground penetrating radar (GPR). An example is presented in which five hypothetical bulk average dielectric constant values were measured with a GPR. Depth specific values of the dielectric constant were obtained, from which the vertical distribution of moisture content, pore water pressure, hydraulic conductivity, and Darcy velocity were estimated.

INTRODUCTION

Knowledge of the vertical distribution of the volumetric soil moisture content is important for estimating the vertical water flux in soil. GPR has the potential for supplying this type of detailed information, however, most investigations have been limited to estimating the bulk average moisture content (via the bulk average dielectric constant) of the soil (e.g., Hallikainen et al, 1985; Dobson et al., 1985; Wang and Schmugge, 1980; Benedetto and Benedetto, 2002; Powers, 1997). Several studies have estimated vertically distributed moisture content using the cross-borehole GPR method (e.g., Kowalsky and Rubin, 2002; Eppstein and Dougherty). However, a disadvantage of the method is that boreholes need to be installed, within which are installed unscreened tubing made of a special material.

To implement measurements using surface GPR, a procedure is needed to extract depth-specific values of the dielectric constant from bulk average dielectric constant.

METHODOLOGY

This section describes an inverse procedure for estimating the depth-specific dielectric constant derived from several bulk average dielectric constant values measured by surface GPR. With a minimum of soil information and the bulk average dielectric constant values, the vertical distribution of moisture content, pore water pressure, hydraulic conductivity and the Darcy flux can be calculated. Only soil type (i.e., percent sand, silt and clay) and bulk density are needed, from which other required soil parameters (porosity, residual moisture content, and water retention parameters) are estimated using the Rosetta neural network (Schaap and van Genuchten, 1998).

The vertically distributed dielectric constant can be obtained from the following equations:

$$\xi_1 = \xi'_1 \tag{1a}$$

$$\xi_n = \frac{\xi'_n \cdot z_n - \sum_{i=1}^{n-1} \xi_i \cdot \Delta z_i}{\Delta z_n} \quad n \geq 2 \tag{1b}$$

where

ξ_n = square root of the dielectric constant for the nth vertical interval

ξ'_n = square root of the average dielectric constant between the soil surface and the bottom of the nth vertical interval, obtained by GPR

z_n = depth of soil between surface and the bottom of the nth vertical interval

Δz_n = the thickness of the nth vertical interval

Equation 1b was obtained by using differential signal delay between the vertical intervals. The differential delay has been converted to the square root of the dielectric constant. Figure 1 illustrates how equation 1 can be applied in practice. The example given in Figure 1 shows a case where $n = 5$ and $z_1 = 0.5 z_2$, and $\Delta z_2 = \Delta z_3 = \Delta z_4 = \Delta z_5$. GPR measurements

are associated with reflections from points (represented by circles) of contrasting dielectric constant. The reflections can be produced by inserting metal rods (e.g., re-bar) in the soil at intervals of Δz to a total depth Z . The maximum depth of the reflectors depends on the frequency of the GPR.

It is recommended that a trench be made in the soil to a depth somewhat greater than Z , and the rods be inserted laterally through the side wall of the trench into the footprint area of the GPR. By inserting the metal rods in this way the soil will remain relatively undisturbed. Each GPR measurement provides a value of the bulk average dielectric constant between the soil surface and the depth z_n . The closer Δz_n are to one another, the more accurately equation 1 represents the media. Because equation 1b depends on the value of ξ_1 , estimated from the square root of the bulk average dielectric constant ξ'_1 , it is recommended that z_1 be small (e.g., 10 cm).

Figure 2 shows the soil cross section parallel to the direction of the reflectors, and the trench cross section.

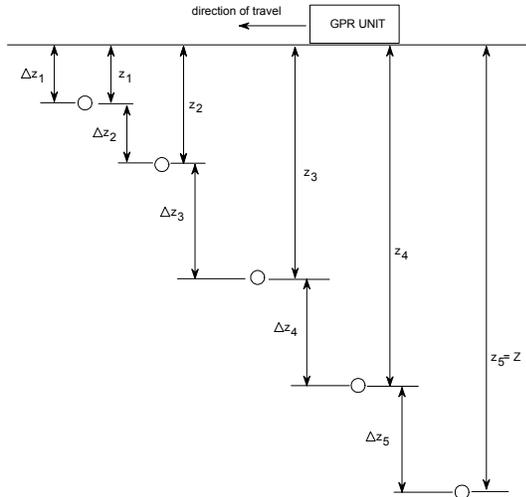


Figure 1. Cross section of soil and reflectors (circles) at five different depths below the surface.

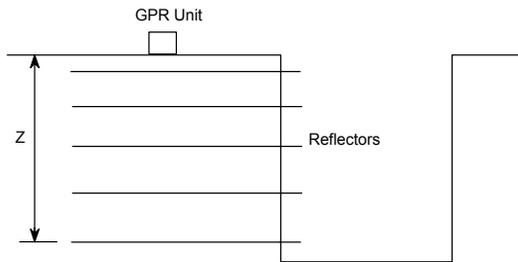


Figure 2. Cross section of reflectors (e.g., re-bars) inserted through the wall of the excavation. GPR unit is moved in a direction normal to the long dimension of the reflectors.

Several dielectric mixing models for relating dielectric constant to moisture content were evaluated for use in this study (Dobson et al., 1985; Wang and Schmugge, 1980; Martinez and Byrnes, 2001; Rial and Han, 2000; Alharthi and Lang, 1987; Shutko and Reutov, 1982). The semiempirical model of Dobson et al. (1985) was the only method considered that accounted for frequency; however, it did not provide accurate estimates of moisture content from the dielectric constant. The Wang and Schmugge (1980) method performed best and was selected for use in this study. In this model, the parameter, W_t , is defined as the transition moisture content at which the dielectric constant increases steeply with increasing moisture content. Consequently two equations ($\epsilon_1(\theta_v)$, and $\epsilon_2(\theta_v)$) are necessary to define the moisture content/dielectric relationship within the moisture content range of 0 to 0.5:

$$\epsilon_1(\theta_v) = \theta_v \cdot \epsilon_{x1}(\theta_v) + (\phi - \theta_v) \cdot \epsilon_a + (1 - \phi) \cdot \epsilon_s \quad \theta_v \leq W_t \quad (2)$$

with

$$\epsilon_{x1}(\theta_v) = \epsilon_i + (\epsilon_w - \epsilon_i) \cdot \frac{\theta_v}{W_t} \cdot \gamma \quad (3)$$

$$\gamma = -0.57 \cdot WP + 0.481 \quad (4)$$

$$WP = 0.06774 - 0.00064 \cdot S + 0.00478 \cdot C \quad (5)$$

$$\epsilon_2(\theta_v) = W_t \cdot \epsilon_{x2} + (\theta_v - W_t) \cdot \epsilon_w + (\phi - \theta_v) \cdot \epsilon_a + (1 - \phi) \cdot \epsilon_s \quad \theta_v > W_t \quad (6)$$

with

$$\epsilon_{x2} = \epsilon_i + (\epsilon_w - \epsilon_i) \cdot \gamma \quad (7)$$

$$W_t(S, C) = 0.49 \cdot WP + 0.165 \quad (8)$$

where
 θ_v = volumetric moisture content
 ϵ_1 = dielectric constant for moisture content less than or equal to W_t
 ϵ_2 = dielectric constant for moisture content greater than W_t
 ϵ_a = dielectric constant of air (1)
 ϵ_i = dielectric constant of ice (3.2)
 ϵ_w = dielectric constant of pure water (81)
 ϕ = porosity
 WP = moisture content at the wilting point (pore water pressure = 15 bars)
 S = sand content in percent of dry soil
 C = clay content in percent of dry soil

W_t = transition moisture content at which the dielectric constant increases steeply with increasing moisture content
 γ = fitting parameter which is related to WP.

Figure 3 shows a comparison of equations 2 and 6 with measured data for Construction Sand and a Miller Clay (Newton, 1977). The construction sand data were obtained as a part of this study. Dielectric constant values were measured using a Model 1502 Tektronix Time Domain Reflectometer (TDR) with a 5.1 cm long wave guide. Soil moisture content values were obtained by the gravimetric method.

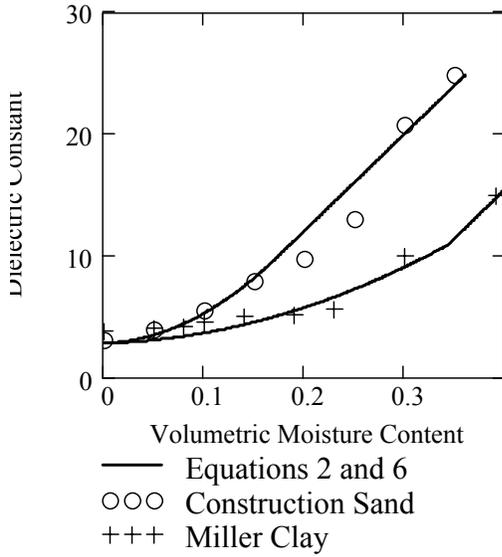


Figure 3. Comparison of measured and calculated dielectric constant for differing volumetric soil moisture content for a Construction Sand and Miller Clay.

The vertical distribution of the pore water pressure and hydraulic conductivity are estimated using the well-known van Genuchten equations (Simunek and van Genuchten, 1999):

$$h(\theta_v) = \frac{\left[\left(\frac{\phi - \theta_r}{\theta_v - \theta_r} \right)^{\frac{1}{m}} - 1 \right]^{\frac{1}{n}}}{\alpha} \quad (9)$$

$$K(\theta_v) = K_s \cdot \left(\frac{\theta_v - \theta_r}{\phi - \theta_r} \right)^{0.5} \cdot \left[1 - \left[1 - \left(\frac{\theta_v - \theta_r}{\phi - \theta_r} \right)^{\frac{1}{m}} \right]^2 \right] \quad (10)$$

$h(\theta_v)$ = pore water pressure
 $K(\theta_v)$ = variably saturated hydraulic conductivity
 K_s = saturated hydraulic conductivity
 θ_r = residual moisture content. Water held at temperatures up to 105°C referred to as hygroscopic water (soil tension of 31 bars) and is virtually part of the mineral structure of the soil (Wang and Schmugge, 1980).
 α = inverse of the air-entry value (or bubbling pressure)
 n = pore size distribution index
 $m = 1 - n^{-1}$ $n > 1$

The Darcy flux is calculated with the Darcy Buckingham equation (Selker et al., 1999):

$$q(z) = K(\theta_v) \cdot \frac{d}{dz} h(\theta_v) \quad (11)$$

Equation 11 can be used to assess the instantaneous flux of water toward the soil surface, or towards the phreatic surface.

RESULTS AND DISCUSSION

Inversion Procedure to Obtain the Dielectric Constant with Depth

The inversion procedure is illustrated using hypothetical values of the bulk average dielectric constant, typical of values that might be measured using surface GPR. Based on the geometrical configuration shown in Figure 1, we assume that $z_1 = 10$ cm, and $\Delta z_2 = \Delta z_3 = \Delta z_4 = \Delta z_5 = 20$ cm. The total depth $Z = 80$ cm. Figure 4 shows the hypothetical vertical distribution of the bulk average dielectric constant and the depth-specific dielectric constant calculated using equation 1. All calculations in this section were made using the software Mathcad (Mathsoft, 2000). Note that in Figure 1, the bulk average value of the dielectric constant (O) have been placed at the depth $z/2$, where z is the associated reflector depth. The depth-specific values of the dielectric constant () are placed at the midpoints of their respective Δz .

Application of the Dielectric Mixing Model

An experimental example is given here for a soil consisting of 20% sand, 30% silt, and 50% clay, and has a bulk density of 1.3 gm/cm³. The soil is classified as clay and is typical of the soil found in Puerto Rico. Using the textural information, bulk density and the calculated wilting point (from equation 5, WP = 0.31), the Rosetta neural network gave the following estimates of θ_r , ϕ , α , n and K_s , respectively: 0.1 cm³/cm³, 0.5 cm³/cm³, 0.0168 cm⁻¹, 1.326 and 18.63 cm/day. Figure 5 shows the vertical distribution of moisture content with depth obtained

from equations 2 and 6. To solve for the moisture content as a function of the dielectric constant, a root search algorithm was used in Mathcad. The distribution of moisture content, as shown in Figure 5, could be produced, for example, by a subsurface drip irrigation line located 30 cm below the surface. Note that at this depth, the moisture content is equal to the porosity (0.5) and is therefore completely saturated.

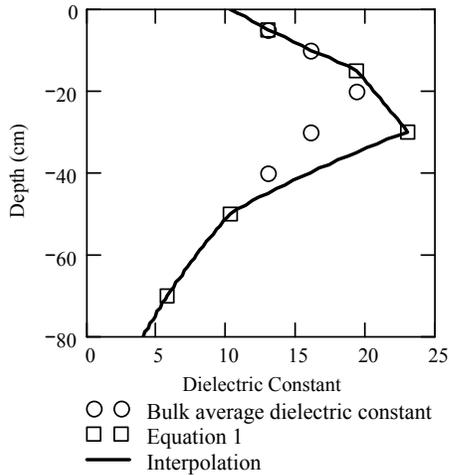


Figure 4. GPR-measured (hypothetical) bulk average dielectric constant (O), depth specific dielectric constant from equation 1 (□), and the interpolated values of dielectric constant (—), used to estimate moisture content, pore water pressure, hydraulic conductivity and Darcy flux.

Estimation of Hydraulic Parameters with Depth

The pore water pressure, hydraulic conductivity and the Darcy velocity were calculated based on the vertical moisture content distribution and the same soil characteristics as given previously. Figure 6, 7 and 8 show the vertically distributed pore water pressure, hydraulic conductivity and Darcy flux calculated using equations 9, 10 and 11, respectively. Figure 7 also shows the distribution of the hydraulic conductivity for comparison. Note that between the soil surface and the 30 cm depth, the flux was upward (i.e., negative). Below the 30 cm depth, the flux was downward (i.e., positive).

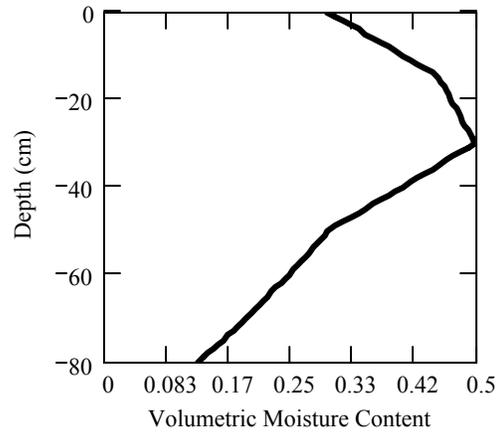


Figure 5. Volumetric moisture content from equations 2 and 6 versus depth.

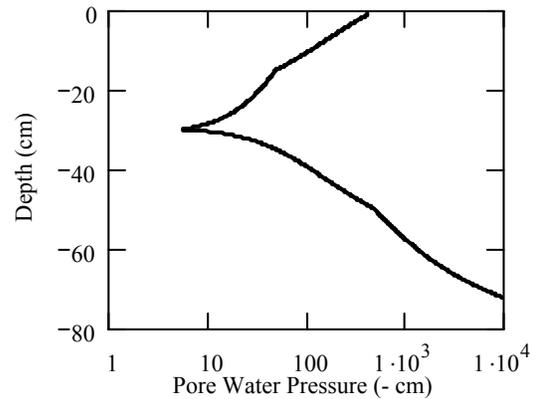


Figure 6. Pore water pressure

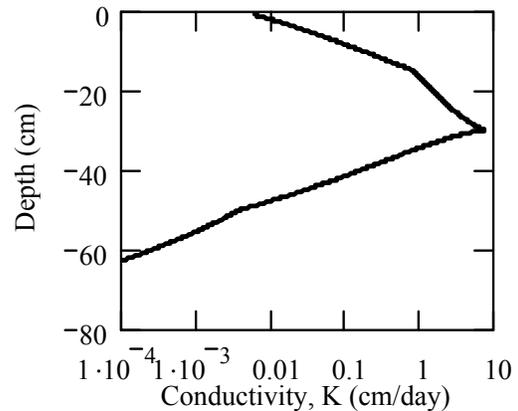


Figure 7. Hydraulic conductivity with depth

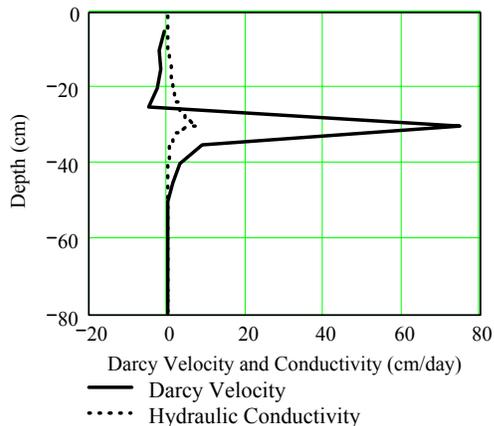


Figure 8. Darcy velocity and hydraulic conductivity with depth

Summary and Conclusions

A procedure was presented for estimating the depth-specific dielectric constant from several GPR-measured bulk average dielectric constants. This approach has an advantage over the borehole GPR approach, which requires the installation of boreholes and the use of special material for the borehole tubing. A disadvantage of the method presented in this paper is the need to install the reflectors, which limits the vertical extent to which measurements can be made. An example was presented in which five hypothetical bulk average dielectric constant values were measured with a GPR. Depth specific values of the dielectric constant were obtained, from which the vertical distribution of moisture content, pore water pressure, hydraulic conductivity, and the Darcy flux were estimated. Additional experiments are planned to verify the procedure under field conditions.

REFERENCES

Alharthi, A. and J. Lange, 1987. Soil water saturation: Dielectric determination. *Water Resources Research*, Vol. 23, No. 4:591-595.

Benedetto, A. and F. Benedetto, 2002. GPR experimental evaluation of subgrade soil characteristics for rehabilitation of roads. Ninth International Conference on Ground Penetrating Radar, Steven K. Koppenjan, Hua Lee, Editors. *Proceedings of SPIE Vol. 4758: 605-60708-714*.

Dobson, M. C., F. T. Ulaby, M. T. Hallikainen, and M. A. El-Rayes, 1985. Microwave dielectric behavior of wet soil-Part II: Dielectric mixing models. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. GE-23, No. 1: 35-46.

Epptein, M. J. and D. E. Dougherty, 1998. Efficient three-dimensional data inversion: Soil characterization and moisture monitoring from cross-well ground-penetrating radar at a

Hallikainen, M. T., F. T. Ulaby, M. C. Dobson, M. A. El-Rayes and L. Wu. 1985. Microwave dielectric behavior of wet soil-Part I: Empirical models and experimental observations. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. GE-23, No. 1: 24-46.

Kowalsky, M. B. and Yoram Rubin, 2002. Suitability of GPR for characterizing variably saturated sediments during transient flow. Ninth International Conference on Ground Penetrating Radar, Steven K. Koppenjan, Hua Lee, Editors. *Proceedings of SPIE Vol. 4758: 605-608*.

Martinez, A. and A. P. Byrnes, 2001. Modeling dielectric-constant values of geologic materials: An aid to ground-penetrating radar data collection and interpretation. *Current Research in Earth Sciences, Bulletin 247, part 1* (<http://www.kgs.ukans.edu/Current/2001/martinez/martinez1.html>)

Rial, W. S. and Y. J. Han, 2000. Assessing soil water content using complex permittivity. *Transactions of the American Society of Agricultural Engineers*, Vol. 43(6)1979-1985.

Schaap, M. G. and van Genuchten, M. Th., 1998. Neural network analysis for hierarchical prediction of soil water retention and saturated hydraulic conductivity. *Soil Sci. Soc. Am. J.* 62:847-855.

Selker, J. S., C. K. Keller and J. T. McCord, 1999. *Vadose Zone Processes*. Lewis Publishers, Boca Raton, USA.

Shutko, A., M. and E. M. Reutov, 1982. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. GE-20, NO. 1:29-32.

Wang, J. R and T. J. Schumgge, 1980. An empirical model for the complex dielectric permittivity of soils as a function of water content. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. GE-18, No. 4:288-295.