

PROBLEMAS PARA LA CLASE DEL 20 DE FEBRERO DEL 2008

Problema 1

Marketing estimates that a new instrument for the analysis of soil samples will be very successful, moderately successful, or unsuccessful, with probabilities 0.3, 0.6, and 0.1, respectively. The yearly revenue associated with a very successful, moderately successful, or unsuccessful product is \$10 million, \$5 million, and \$1 million, respectively. Let the random variable X denote the yearly revenue of the product. Determine the mean and variance of the random variable

Mean and variance for exercise 3-19

$$\begin{aligned}\mu &= E(X) = 10f(10) + 5f(5) + 1f(1) \\ &= 10(0.3) + 5(0.6) + 1(0.1) \\ &= 6.1 \text{ million}\end{aligned}$$

$$\begin{aligned}V(X) &= 10^2 f(10) + 5^2 f(5) + 1^2 f(1) - \mu^2 \\ &= 10^2 (0.3) + 5^2 (0.6) + 1^2 (0.1) - 6.1^2 \\ &= 7.89 \text{ million}^2\end{aligned}$$

Problema 2

Pepitos industry contracts for a wide variety of services, such as vehicle maintenance and mess hall operations. In administering these contracts, inspections are required of units randomly selected from the defined lot (statistical population). One food service contract stipulates an acceptable quality level (AQL) for equipment cleanliness of 6.5% during meal preparations. The allowable proportion of unclean equipment is then 0.065.

Based on the above AQL, the Pepitos Industry requires that a sample size of $n=13$ be used and that the lot be accepted or rejected according to the following decision rule for the observed number of defectives R (meal times during which some dirty equipment is found).

Accept a lot if $R \leq 2$

Reject a lot if $R > 2$

What is the probability of accepting the lot?

X: número de equipos que resultan sucios durante meal times.

Se acepta el lote si tiene 2 o menos equipos sucios.

La porción de equipos sucios es $p = 0.065$ y es constante

Parámetros dados: $p = 0.065$ y $n = 13$

Encuentro que solo hay dos opciones, o esta limpio o esta sucio

Me dan el numero de éxitos en n intentos ya que me dicen que con 2 equipos sucios o menos puedo aceptar el lote. De igual manera me dicen que con mas de dos 2 equipos sucios lo debo rechazar.

Se utiliza entonces la distribución binomial

$$f(x \leq 2) = \binom{13}{0} 0.065^0 (1-0.065)^{13-0} + \binom{13}{1} 0.065^1 (1-0.065)^{13-1} + \binom{13}{2} 0.065^2 (1-0.065)^{13-2}$$
$$= 0.4174 + 0.3772 + 0.1573 = 0.9519$$

Problema 3

Many time-sharing computer systems operate near their capacity, so users often can't access the central processor because all teleports are busy. Consider the number of attempts K a client must make to gain a connection. Each call may be viewed as a trial having "busy signal" or "connection" as outcomes. Assuming the system is busy 95% of the time, the probability that exactly 5 calls are required for a connection is? How many calls a user can expect to place in order to get connection?

X: Número de llamadas para tener conexión

Me dan solo dos posibles outcomes o esta ocupado o hay conexión y la probabilidad de que haya un éxito es constante de 0.005 por lo tanto son intentos bernoulli

Me dan una proporción p de 0.95 de que la llamada obtenga un signo de ocupado.

Se utiliza entonces una distribución geométrica

The probability for a call to complete is 0.05

The probability that a call gets busy signal is 0.95

$f(x) = (1-p)^{x-1} p = (1-0.05)^{5-1} 0.05 = 0.041$ es la probabilidad de que se requieran 5 llamadas para tener conexión.

$E(x) = \frac{1}{p} = \frac{1}{0.05} = 20$ Llamadas. Se deben hacer 20 llamadas en promedio para obtener conexión. Por esto la probabilidad de obtener conexión con solo 5 intentos de llamada es tan baja

Problema 4

From a lot of 10 parts, 4 are selected at random and inspected. If the lot contains 3 defective parts. What is the probability that all 4 will be conforming parts?

X: número de partes conformes en la muestra n

Me dan un tamaño de lote y un tamaño de muestra sacada de ese lote

Me dan el número de éxitos considerando como éxito que las partes sean conformes

Los elementos se consideran o éxito o fracaso

N = tamaño de lote de 10

n = tamaño de muestra de 4 partes

K = 10 - 3 = 7 partes conformes en todo el lote

N-K = fracasos = 10-7 = 3 partes no conformes en todo el lote

$$f(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}} \quad f(4) = \frac{\binom{7}{4} \binom{10-7}{4-4}}{\binom{10}{4}} = \frac{35 \cdot 1}{210} = 0.1666$$

Problema 5

In an inventory study it was determined that on the average, the demands for a particular item at a warehouse were five times per day. What is the probability that on a given day this item is not request? What is the probability that the item is requested more than 2 times in a given day? What is the probability that the item is requested 2 or more times in a given day?

X: tasa que me preguntan

Me dan una tasa que seria la demanda en un día de un artículo

Se aplica la distribución Poisson

$$\lambda = 5$$

$$x = 0$$

La probabilidad de que en un día dado el item no sea requerido es:

$$p(0;5) = \frac{e^{-5}5^0}{0!} = e^{-5} = 0.00067$$

La probabilidad de que el item sea requerido más de 2 veces en un día dado es:

$$1 - p(x \leq 2)$$

$$p(x \leq 2) = p(0;5) + p(1;5) + p(2;5)$$

$$p(x \leq 2) = \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!}$$

$$p(x \leq 2) = 0.00067 + 0.034 + 0.084 = 0.125$$

$$1 - p(x \leq 2) = 1 - 0.125 = 0.875$$

La probabilidad de que el item sea requerido 2 o más veces en un día dado es:

$$1 - p(x \leq 1)$$

$$p(x \leq 1) = p(0;5) + p(1;5) = 0.00067 + 0.034 = 0.040$$

Problema 6

A trading company has eight computers that it uses to trade on the New York Stock Exchange (NYSE). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening and each day is an independent trial.

- (a) What is the probability that all eight computers fail in a day?
- (b) What is the mean number of days until a specific computer fails?
- (c) What is the mean number of days until all eight computers fail in the same day?

X: número de computadores que fallan

Éxitos: son catalogados como los fallos

Los fallos son independientes y cada día es un evento independiente

Tengo $n = 8$ computadores y la probabilidad de que un computador falle en un día es de $p = 0.005$

$$(a) P(X = 8) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{8}{8} 0.005^8 (1-0.005)^{8-8} = 3.91 \times 10^{-19}$$

Para los puntos b y c la variable aleatoria X se define como: número de días o intentos hasta que se da un éxito o fallo por lo tanto se utiliza la distribución geométrica

$$(b) E(x) = \frac{1}{p} = \frac{1}{0.005} = 200 \text{ días hasta que se da el primer fallo}$$

$$(c) E(x) = \frac{1}{p} = \frac{1}{3.91 \times 10^{-19}} = 2.56 \times 10^{18} \text{ días hasta que se da que los 8 computadores fallen}$$